The Emergence of a Major Invoicing Currency

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Very Preliminary. Please do not circulate.
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Abstract

This paper proposes a model to explain how the currency of a country evolves in importance as an invoicing currency in trade settlement as the relative size of the country grows. The coalescing effect, which induces individual firms to choose the similar invoicing currency mix as their competitors so as to limit output volatility, not only facilitates the internationalization of a currency, but also accelerates this process by creating a positive feedback between actions of individual firms and their competitors. We also show that, as the country grows, its currency is more likely to be used by small and close trading partners to invoice trade than its larger and less close counterparts. Moreover, wider use of a currency for invoicing in the issuing country's home market would help spread the use of the currency to other markets. Furthermore, we can explain the "tipping phenomenon" — the non-linear relationship between the international use of a currency and the economic size of the issuing country. The non-linearity arises from an affinity for the established international currency. We confront our hypotheses with trade invoicing data on euro and dollar of 35 countries (1998-2010) and invoicing data on four major international currencies for trade between Thailand and 19 countries (2001-2008), and find that the results are consistent with our theory.

Keywords: currency, invoicing, vehicle currency, currency internationalization, pass-through

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1 Introduction

This paper proposes a theory to explain how the currency of a country emerges as an international currency in trade settlement and then confronts the theory with data. In principle, this currency can be any currency. However, we do have in mind the Chinese renminbi (hereinafter RMB) when we write the paper.

Interests in the potential of the RMB to become internationalized is fueled by the rise of China, the mounting external debts of the US as well as the unresolved European debt crisis. The RMB is currently not qualified as an international currency because China's underdeveloped finance impedes China to develop a thick market, a precondition for a currency to become widely used. Nevertheless, as is well known, China also enjoys exceptional advantages in making its currency internationalized: as the second largest economy in the world, it has the scale necessary in order to create deep and liquid markets. The sheer volumes of its foreign trade and inward foreign direct investment create a large installed base for RMB-based transactions. Besides, since the eruption of the global economic crisis in 2008-2009, the supreme position of the U.S. dollar as a global medium of exchange and a reserve currency is eroded; by contrast, China's continuing robust growth in the last three decades inspires worldwide confidence in China as a growth engine capable of leading the recovery of the global economy. Therefore, the RMB has great potential to become internationalized in the sense that a global role of currency should be commensurate with its home country's huge economic might in the long run, as indicated by many previous studies on internationalization of currencies (e.g. Krugman 1984, Eichengreen 2011).

However, a more important question for Chinese and other countries' policy makers is: how would the dynamic course of the internationalization look like, assuming China can keep its current growth and the Chinese authorities set their minds to overcome their disadvantages step by step? Looking back at the history of the replacement of the sterling by the U.S. dollar, the most noticeable pattern may be a slow pace of increase in the adoption of the new currency in international trade invoicing, followed by a sharp rise later. The dollar in 1914, like the RMB today, played a negligible role in the world, although the United States was the largest trading nation by then. In the years after the First World War, this trend continues with sterling playing an important role in global trade settlement, despite the decline of Britain as a global economic power. However, a dramatic change follows: after the Second World War the collapse of sterling accelerated and the dollar replace sterling as the leading international currency.

The above paragraph describes how one currency (the US dollar) evolved to become a dominant reserve currency over time. It was observed that the time trajectory of the reserve share of the currency was highly non-linear with respect to the economic size of the country as the country's share of global GDP grew. Interestingly, when comparing across countries at any given time, we also observe such non-linearity. That is, when examining the reserve shares of the five major international currencies in the world nowadays — the U.S. dollar, the euro, the British pound sterling, the Japanese yen and the Swiss franc — we observe that the responsiveness of reserve share of a currency to changes in the determinant variables (such as global GDP share of the country) is much smaller when the reserve share is low than that when the reserve share is high. Consequently, the top reserve currency attains a disproportionately high reserve share relative to its GDP share. This convex relationship between the share of a country's currency in central banks' reserves and its economic size is called the "tipping phenomenon".

Krugman (1984) suggests a non-linear curve that depicts the relationship between the desired use of an international currency and its actual use. The desired use of a currency increases in its actual use, with the slope increasing when the actual use is low and decreasing when the actual use is high. This relationship would be very helpful for investigating the course of RMB's global use as an invoicing currency if we can understand the underlying rationale, which, unfortunately, is not provided by Krugman's paper.

As we review the literature, we found that there is a shortage of theories that explain the process of internationalization of a currency, in sharp contrast to the relatively richer body of empirical studies. Matsuyama et al (2001) and Rey (2001) are two notable exceptions. Matsuyama et al (2001) use the framework of random matching games to discuss whether and how a national currency become circulated internationally under different scenarios in a two-country model. They also point out that the use of money as medium of exchange hinges critically on a strategic externality and economies of scale and therefore the currency of a larger country has higher chance to be accepted as an international currency. Rey (2001) seeks to explain the rise and fall of the pound and the dollar as vehicle currencies with a cash-in-advance model in an open economy framework where explicit transaction technology is added.

The common focal point of both papers is the lubricating function of a currency, i.e. the medium of exchange to lower transaction costs. However, this may not be the most important factors, at least not the only one, to determine how internationalized a currency can go, as indicated by Goldberg and Tille (2008). They show with quantitative results both from model and empirical tests that transaction cost is a much minor factor compared to the "coalescing effect" which means firms are motivated to follow the invoicing currency choice of its competitors in the market to minimize their price volatilities relative to others. This effect plays an essential role in limiting the fluctuations of profit flows of a firm, especially when prices are not adjusted frequently.

The goal of this paper is to construct a theory to explain how a country's currency becomes more heavily used as an invoicing currency as the country's economic size increases, and then test the theory. In particular, we extend the model of Goldberg and Tille (2008) to a model with multiple countries and multiple currencies. The main intuition is that firms would like to coalesce to use the mainstream currency in the market to invoice and settle to minimize their price volatility relative to the market. When one of its foreign markets has grown, exporters have to put more weight on the price index of this market and have to invoice a higher fraction of their exports to this market in the local currency. When every firm chooses to do so, the invoicing share of this currency in markets other than this market would also rise accordingly so as to limit goods arbitrage.

Furthermore, as a market grows relative to the rest of the world, firms from different countries adjust the compositions of their invoicing currencies at different paces: firms in small and nearby trading partners, which usually have smaller domestic markets and rely more heavily on selling to this growing market, are more sensitive to the change, and they adjust the compositions of their invoicing currency faster than their counterparts in larger trading partners. Thus, there is larger impact of the growing market on smaller and nearer countries.

The above two paragraphs describe two channels affecting the increasing use of a currency in trade invoicing: one is the direct channel which means the home country's larger size implies the home country's higher share in the sales of exporters and hence the higher share of its currency being used abroad; the other is the indirect channel, which is the spillover of the currency use from this growing market to other markets. These two channels will both contribute to the "tipping phenomenon": the impact of the direct channel is relatively weak when the rising country is still small due to the dilution of trade costs. In addition, the self-justification of established vehicle currencies would be a bigger obstacle to the wide circulation of the new vehicle currency when its home country is smaller. However, the direct effect will get stronger as the country grows relative to the rest of the world. The indirect channel also gets relatively stronger when the country is larger because the strengthened direct channel widens the gap between the use of its currency in its home market and in other markets. This wider gap implies that each exporter has to adjust the composition of its invoicing currencies by a larger degree for the same shift in the shares of markets in sales. Both channels give rise to an increasing but convex relationship between the share of the global use of the currency and the world GDP share of its home country.

We confront the theoretical implications with data on the use of euro and dollar as invoicing currency in exports and imports in 35 countries (1998-2010) and data on the use of four major international currencies — U.S. dollar, the euro, British pound sterling and Japanese yen in Thailand's trade (2001-2008). The empirical results are not only consistent with our theoretical predictions, but also show the importance of the mechanisms we emphasize.

The remainder of the paper is organized as follows: Section 2 presents a model with multiple countries and multiple currencies. We compare the choice of the composition of invoicing currencies by firms in countries of different sizes. Based on this analysis we compute the trajectory of the global use of a currency as the issuing country grows relative to the rest of the world. Section 3 tests the empirical implications of the model by confronting it with data on invoicing currencies from different countries. Section 4 concludes the paper.

2 A Three-Country / Two-Market / Two-Currency Model of Invoicing

This section develops a model of vehicle currency that is inspired by Goldberg and Tille (2008). Our model has two major differences from theirs: (i) firms export to multiple markets; (ii) we consider a Nash equilibrium where each exporter makes an optimal invoicing-currency decision given the invoicing-currency decisions of all other exporters. We describe the environment in section IIA and solve the model in section IIB. In section IIIC and IIID we discuss how a currency becomes more widely used as a vehicle currency for trade settlement as the issuing country grows relative to the rest of the world.

2.1 The Setup of the Model

We consider a world with three countries — a large country A, a growing country B and a small country C. The sizes of country B and C are initially small relative to country A. And the size of country B keeps growing while the sizes of country A and C remain unchanged. The local currencies of the three countries are denoted by a, b and c respectively. We assume that there is one single vehicle currency in the beginning, currency a, and we focus on the emergence of currency b as another vehicle currency as country B grows. We always ignore the possibility of currency c as a candidate for international currency due to C's small size.

The markets in the world can be divided into two: market A' and market B. Market A' is country A plus country C (and possibly other small countries whose size are also ignorable). It is mainly dominated by a, at least when B does not dominate over A. In contrast, market B is initially dominated by a but possibly occupied by b during B's growth. An exporting firm's owner needs to decide which currencies to invoice her products. Specifically, an exporter in country E ($E \in \{A, B, C\}$), produces a brand z and sells it in the two markets. She posts her price $P_E^k(z)$ in currency k_E before knowing the realization of various shocks affecting the economy. The invoicing currency k_E can be a basket of currency a and b, with a share of β_E on b (and therefore $1 - \beta_E$ on a). At the aggregate level, we assume that a share of $\eta_{A'}$ of the aggregate price $P_{A'}$ is invoiced in currency b in market A' while a share of η_B of the aggregate price P_B is invoiced in currency b in market B. We assume $\eta_B > \eta_{A'}$ and will show later that it is true in equilibrium. The variables β_E and η_D ($D \in \{A', B\}$) can be understood as the partial pass-through of exchange rate to the price.[1],[2] γ_E is the share of market B in the total sales of a country-E firm in steady state. Note that all variables β, γ, and η pertain to either currency b or market B without explicitly indicating so.

Note that the price $P_E^k(z)$ and the invoicing currency k_E set by a firm are applied to both of its markets. And we assume that the invoicing currency is also the one actually used in the payment.[3] These assumptions eliminate the possibility of good arbitrage in principle.[4]

[1] From now on, we use E as a general notation for exporting countries and D for destination market. Also, we use e to denote the currency of the exporting country.

[2] This way to present a partial pass-through of exchange rate fluctuations to consumer prices is standard in literature Corsetti and Pesenti (2005, 2002), Engel (2006), Goldberg and Tille (2008).

[3] Krugman (1984), Friberg and Wilander (2007)

[4] It can also be relaxed without hurting the essence of our model. If we assume that a firm can invoice in different currencies for different markets, say, in currency k_1 and k_2 for market A' and B respectively, the prices in the two markets will be practically different after the realization of exchange rates. Therefore, given the volatility of the exchange rates, a firm should make her baskets of k_1 and k_2 not deviate from each other too

Firms face monopolistic competition in both markets and the demand functions are:

$$Y_{EA'} = \left(\frac{S_{ek_E} T_{EA'} P_E^k}{S_{ea} P_{A'}}\right)^{-\lambda} C_{A'} \text{ and } Y_{EB} = \left(\frac{S_{ek_E} T_{EB} P_E^k}{S_{eb} P_B}\right)^{-\lambda} C_B$$

where $P_{A'}$ and P_B are the price indices across all brands on both markets denominated in their local currencies; S_{ea} is the exchange rate between currency e (the currency of the exporting country) and currency a with an increase corresponding to a depreciation of currency e; $C_{A'}$ and C_B are the (exogenous) aggregate demands across all brands in market A' and B; $T_{EA'}$ and T_{EB} capture a variety of exogenous trade frictions that can distort the prices faced by consumers in the two markets. $\lambda > 1$ is the price-elasticity of demand.

The meanings of the trade costs $T_{EA'}$ and T_{EB} here are richer than one might think. There are at least two types of elements involved: One is the traditional trade costs. Countries differ in trade costs due to transportation costs, trading bloc memberships or extents of economic integration. This will make the breadth and depth of the circulation of a vehicle currency substantially vary across countries. Another is factors that are associated with a currency's international role. Given prices and exchange rates, consumers prefer to use currencies that are widely used.[5] Wright and Trejos (2001) shows that a currency gains additional value if it circulates abroad. Established vehicle currencies are also usually freely convertible at lower transaction costs. These components of self-justification and self-reinforcement are particularly important to newly internationalized currencies.

We assume that $T_{AA'}^{-\lambda} T_{AB}^{\lambda} > T_{CA'}^{-\lambda} T_{CB}^{\lambda} > T_{BA'}^{-\lambda} T_{BB}^{\lambda}$. It means that with normalized trade costs in market B, country A is more advantageous than country B in market A' while country C is just in between. For convenience of presentation, we let $T_{EA'}^{-\lambda} T_{EB}^{\lambda} = T_E$, thus the assumption becomes $T_A > T_C > T_B$.

The technology for production is decreasing returns to scale with the degree of returns to scale $0 < \alpha < 1$:

$$Y_E = \frac{1}{\alpha}(H_E)^{\alpha}$$

where H_E is a composite input with a unit cost of W_E in her local currency.

far to limit the potential of arbitrage. An simply policy for a firm is $k_2 = k_1 + k_0$ where k_0 is a constant which reflects the tolerance of the producer on its expected price difference ex ante.

[5] Wright and Trejos (2001) shows that a currency gains additional value if it circulates abroad.

2.2 The invoicing-currency choice of a firm

First we determine the invoicing currency choice of a typical firm in each country. We deal with the problem of an exporter from country C; those of firms from countres A and B are just analogous.

An exporter in C sets her price P_C^k in currency k_C to maximize her discounted expected profits:[6]

$$\begin{aligned}
\max_{P_C^k, \beta_C} \Pi_C &= Ed_C[S_{ck_C} P_C^k (Y_{CA'} + Y_{CB}) - W_C \alpha^{\frac{1}{\alpha}} (Y_{CA'} + Y_{CB})^{\frac{1}{\alpha}}] \\
&= Ed_C \left\{ S_{ck_C} P_C^k \left[\left(\frac{S_{ck_C} T_{CA'} P_C^k}{S_{ca} P_{A'}} \right)^{-\lambda} C_{A'} + \left(\frac{S_{ck_C} T_{CB} P_C^k}{S_{cb} P_B} \right)^{-\lambda} C_B \right] \right. \\
&\quad \left. - W_C \alpha^{\frac{1}{\alpha}} \left[\left(\frac{S_{ck_C} T_{CA'} P_C^k}{S_{ca} P_{A'}} \right)^{-\lambda} C_{A'} + \left(\frac{S_{ck_C} T_{CB} P_C^k}{S_{cb} P_B} \right)^{-\lambda} C_B \right]^{\frac{1}{\alpha}} \right\}
\end{aligned}$$

where d_C is the state-specific discount factor which is independent of the profits of a particular firm.

The maximization problem is difficult to solve in general. We approximate the logarithm of the profit function to second order around a steady state where the economy is not affected by any shock.[7] In steady state the choice of invoicing currency would be irrelevant because all currencies are equivalent to each other given that exchange rates are fixed. What matters is only the pricing decision. With this independence, we can separate the maximization over P_C^k and over β_C and obtain the following solution for β_C (refer to Appendix 5.1 for details):

$$\beta_C = \Omega[(1 - \gamma_C)\eta_{A'} + \gamma_C \eta_B] + (1 - \Omega)\rho_C$$

where γ_C is the share of market B in the total sales of a country-C firm in steady state; $\rho_C = \frac{E(m_C s_{ab})}{E(s_{ab})^2} + \frac{E(s_{ac} s_{ab})}{E(s_{ab})^2}$ where $m_C = \frac{1-\alpha}{\alpha}[(1-\gamma_C)c_{A'} + \gamma_C c_B] + w_C$. Lower case letters denote logs, namely, $c_{A'} = \log(C_{A'})$, $c_B = \log(C_B)$ and $w_C = \log(W_C)$. The variables $\Omega = \frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+\alpha}$ and $1 - \Omega$ are the weights assigned to the two factors determining the choice of invoicing currency. The intuition is discussed below:

The first factor is the "coalescing effect", captured by the first term, which means that an exporter has an incentive to follow the invoicing strategy of its competitors in order to

[6] The z index on price $P_{Ck}(z)$ can be ignored because of the homogeneity among firm within one country.

[7] We follow the method of Goldberg and Tille (2008). Bachetta and van Wincoop (2005) adopt a similar approximation.

limit output volatility, especially when goods are close substitutes (λ is large) or the degree of decreasing returns to scale is stronger (α is small). In this model, the coalescing effect is represented by $[(1-\gamma_C)\eta_{A'} + \gamma_C \eta_B]$. Intuitively, an exporter faces two groups of competitors in each of the two markets, whose invoicing currency policies are $\eta_{A'}$ and η_B as a whole, respectively. To invoice in a common currency mix in all markets, the weights on the two groups should be the corresponding sizes of the two export markets. Obviously, the use of an invoicing currency depends on its market size.

The second factor, captured by the second term, is the set of fundamentals reflecting the impacts of macroeconomic volatility as well as exchange rate fluctuations. The macroeconomic volatility is represented by the comovement of the changes of marginal cost m_C and exchange rate s_{ab}, relative to the volatility of exchange rate. The change in m_C is determined either from the change of input cost w_C or from the change of demand $c_{A'}$ or c_B. Thus an exporter prefers to invoice in a currency which delivers a hedging benefit by limiting the deviation between marginal cost and marginal revenue. The exchange rate fluctuation is linked to the exchange rate regime. If s_{ac} partially co-moves with s_{ab}, then $0 < \frac{E(s_{ac}s_{ab})}{E(s_{ab})^2} < 1$. The larger is the weight of currency b, the more intensively the exporters in country C would like to invoice in currency b. A similar logic applies to currency a. In sum, a currency that can help hedge cost or exchange rate movement would be favored as an invoicing currency.

Nonetheless, according to Goldberg and Tille (2008), the second factor is quantitatively minor compared to the first factor, the coalescing effect, for two reasons: one is that the weight on the coalescing effect, Ω, is likely to be large (they give a numerical example that $\alpha = 0.65$ and $\lambda = 6$ lead to $\Omega = 0.76$); the other is that the magnitude of exchange rate fluctuation is much larger than the magnitude of their co-movements with other variables. Therefore, we simply assume that $\frac{E(m_C s_{ab})}{E(s_{ab})^2} \approx 0$. As for the exchange rate fluctuation term $\frac{E(s_{ac}s_{ab})}{E(s_{ab})^2}$, we simply assume it is between 0 and 1. Thus $0 < \rho_C < 1$.

For firms from country A and B, the analysis can be carried out as before. The shares of currency b in invoicing chosen by a typical firm in country A and country B are respectively:

$$\beta_A = \Omega[\gamma_A \eta_B + (1-\gamma_A)\eta_{A'}] + (1-\Omega)\rho_A$$
$$\beta_B = \Omega[\gamma_B \eta_B + (1-\gamma_B)\eta_{A'}] + (1-\Omega)\rho_B$$

where $\rho_A = \frac{E(m_A s_{ab})}{E(s_{ab})^2}$ and $\rho_B = \frac{E(m_B s_{ab})}{E(s_{ab})^2} + 1$. Notice that the impact of exchange rate fluctuations for country A $\frac{E(s_{aa}s_{ab})}{E(s_{ab})^2}$ is 0 while that for country B $\frac{E(s_{ab}s_{ab})}{E(s_{ab})^2}$ is 1. Therefore we have

$\rho_B > \rho_C > \rho_A \approx 0$.

Besides ρ_E, the decisions made by firms from different countries are also different in market B's shares in their total sales. The share of market B in firm E's total sales in steady state is given by $\gamma_E = \frac{S_{ab}^{\lambda} P_B^{\lambda} C_B}{T_E P_{A'}^{\lambda} C_{A'} + S_{ab}^{\lambda} P_B^{\lambda} C_B}$ (Appendix 5.1). Given $T_A > T_C > T_B$, we have $\gamma_A < \gamma_C < \gamma_B$. The share of market B in the total sales of firms from country B is the largest while that for firms from country A is the smallest.

With these differences, it is straightforward to have the following proposition:

Proposition 1 *The firms from the small country C are more likely to use currency b as invoicing currency than those from the large country A, i.e. $\beta_C > \beta_A$.*

This result is quite intuitive. Compared to country A , country C are more reliant on exporting to country B, then their exporters tend to invoice a larger share of their sales in currency b.

2.3 The change of the invoicing currency of a firm as country B grows

Now we assume that country B grows while country A and C remain unchanged in size. As a result, country B's share of world demand, $X_B = \frac{S_{ab}^{\lambda} P_B^{\lambda} C_B}{P_{A'}^{\lambda} C_{A'} + S_{ab}^{\lambda} P_B^{\lambda} C_B}$, increases. Here the demand is "effective"demand faced by exporters in steady state adjusted by price indices which are converted into the same currency.

Proposition 2 *Suppose the aggregate invoicing currency mix in both markets $\eta_{A'}$ and η_B are fixed. (i) As country B grows, the share of currency b as invoicing currency, β_E, will increase. (ii) Moreover, if country B is not too large (i.e. $X_B \le \frac{\sqrt{T_A T_C}}{1+\sqrt{T_A T_C}}$), the share in the small country C will increase more than in the large country A, i.e. $\frac{\partial \beta_C}{\partial X_B} > \frac{\partial \beta_A}{\partial X_B}$.*

The first part of this proposition is a natural result of the coalescing effect. When one of the markets faced by a firm becomes larger, in order to limit the fluctuations in their outputs, exporters put more weight on the local currency of this market. The second part of this proposition inherits the insight of Proposition 1: the rise of X_B has stronger impact on closer trading partners. This conclusion needs the condition that country B is not too large because otherwise the saturation effect will reverse the relative impacts on country C and country A. Is this assumption restrictive? When B is not overwhelmingly large, T_A and T_C are likely to be greater than 1, implying that the upper bound of X_B, $\frac{\sqrt{T_A T_C}}{1+\sqrt{T_A T_C}}$, is greater than $\frac{1}{2}$. This upper bound is large enough for country B to initiate the internationalization of its currency.

We should also note that, the coalescing effect is essential for currency internationalization. If there is no coalescing effect, the only concern of an individual firm is simply the macroeconomic volatility and the exchange rate uncertainty. Hence one firm's decision about the invoicing currency is independent of the decisions of others. Thus, market sizes have no effect on these decisions. The coalescing effect reflects the competition among firms which creates the strategic interactions among them. This leads to the emergence of a major invoicing currency as its issuing country grows in size. With the growth of X_B being the driving force, the coalescing effect is the engine that makes the machine run.

2.4 The change of the aggregate invoicing currencies as country B grows

Now we solve the share of currency b in aggregate invoicing $\eta_{A'}$ and η_B. Let $M_{EA'}$ denote the market share of a typical multinational firm from country E selling in market A' and M_{EB} the market share of a typical multinational firm from country E selling in market B. Besides, market A' and B have sellers who only sell in their own market, whose market shares are denoted by $M^d_{A'}$ and M^d_B respectively.

According to our definition of the aggregate shares of the currency b as invoicing currency in the two markets, $\eta_{A'}$ and η_B should satisfy, in equilibrium, the following equations:

$$\begin{aligned}
\eta_{A'} &= M_{AA'}\beta_A + M_{BA'}\beta_B + M_{CA'}\beta_C + M^d_{A'A'} \cdot 0 \\
\eta_B &= M_{AB}\beta_A + M_{BB}\beta_B + M_{CB}\beta_C + M^d_{BB} \cdot 1
\end{aligned}$$

where $M_{AA'} + M_{BA'} + M_{CA'} + M^d_{A'} = M_{AB} + M_{BB} + M_{CB} + M^d_B = 1$. These two equations also guarantee that each market clears.[8] To simplify our calculation, we assume that the shares occupied by the domestic firms are just the same, i.e. $M^d_{A'} = M^d_B = M^d$.

Substituting individual firms' decisions into the equilibrium conditions, the system of equations that determines $\eta_{A'}$ and η_B can be reduced to:

$$\begin{aligned}
\eta_{A'} &= f_{A'}\eta_{A'} + g_{A'}\eta_B + h_{A'} \\
\eta_B &= f_B\eta_{A'} + g_B\eta_B + h_B
\end{aligned}$$

[8] Another perspective to derive them. See Appendix 5.4

where

$$
\begin{aligned}
f_{A'} &= \Omega[M_{AA'}(1-\gamma_A) + M_{BA'}(1-\gamma_B) + M_{CA'}(1-\gamma_C)] \\
g_{A'} &= \Omega[M_{AA'}\gamma_A + M_{BA'}\gamma_B + M_{CA'}\gamma_C] \\
h_{A'} &= (1-\Omega)(M_{AA'}\rho_A + M_{BA'}\rho_B + M_{CA'}\rho_C) \\
f_B &= \Omega[M_{AB}(1-\gamma_A) + M_{BB}(1-\gamma_B) + M_{CB}(1-\gamma_C)] \\
g_B &= \Omega[M_{AB}\gamma_A + M_{BB}\gamma_B + M_{CB}\gamma_C] \\
h_B &= (1-\Omega)(M_{AB}\rho_A + M_{BB}\rho_B + M_{CB}\rho_C) + M^d.
\end{aligned}
\tag{1}
$$

Therefore $\eta_{A'}$ and η_B can be solved.

The coefficients $f_{A'}$ and f_B can be viewed as the aggregate impacts of the coalescing effect from market A', while $g_{A'}$ and g_B are the counterparts from market B. The variables $h_{A'}$ and h_B are the aggregate impacts from macroeconomic volatility and domestic users.

Proposition 3 *In Nash equilibrium, the aggregate share of currency b as invoicing currency in market B is higher than that in market A', i.e. $\eta_B > \eta_{A'} > 0$.*

Both $\eta_{A'}$ and η_B are surely positive due to the existence of domestic sellers. Their invoicing decisions are never affected by outside changes so that they act as ballasts for their local currencies. $\eta_B > \eta_{A'}$ because on average, market B is more important to exporters of market B than to exporters of market A' in terms of the share in sales.

Next we turn to discuss how the aggregate shares $\eta_{A'}$ and η_B change with X_B. As one can see, the dynamics involve a number of the exogenous variables such as T_E and M_E. Thus discussing analytical results is difficult in this general version of the model. We make the following simplifying assumption under which the model is more tractable and the insight becomes clearer.

To determine $\eta_{A'}$ and η_B, the "sufficient statistics" are just $g_{A'}$, g_B and M^d. We think about an average exporter of market A' which has an average relative trade cost $\tilde{T}_{A'}$ and a similar one of market B with an average relative trade cost $\tilde{T}_B$. Naturally, $\tilde{T}_{A'} > \tilde{T}_B$. According to their definitions, $g_{A'} = \Omega\frac{X_B}{\tilde{T}_{A'}+(1-\tilde{T}_{A'})X_B}$ and $g_B = \Omega\frac{X_B}{\tilde{T}_B+(1-\tilde{T}_B)X_B}$. From now on we only need to consider three parameters: $\tilde{T}_{A'}$, $\tilde{T}_B$ and M^d.

Proposition 4 *As country B grows, the aggregate shares of currency b as invoicing currency in the two markets, $\eta_{A'}$ and η_B, will increase. Moreover, if country B is not too large (i.e.*

$X_B \leq \frac{\sqrt{\tilde{T}_{A'}\tilde{T}_B}}{1+\sqrt{\tilde{T}_{A'}\tilde{T}_B}}$), η_B *increases more than* $\eta_{A'}$, *i.e.* $\frac{\partial \eta_B}{\partial X_B} > \frac{\partial \eta_{A'}}{\partial X_B}$.

This proposition, as the extension of Proposition 2 to Nash equilibrium, formally predicts the internationalization of currency b. The first part of this proposition includes both the direct impact discussed by Proposition 2 and the indirect impact caused by the positive feedback of η_D on β_E: when η_D increases due to the direct impact of X_B on β_E, it will in turn raise β_E further via the coalescing effect. This will in turn increase η_E further. This positive feedback can accelerate the spread of currency b.

As X_B increases, the distortions contributed by the trade barriers become relatively smaller, hence the difference between $g_{A'}$ and g_B is enlarged, which in turn magnifies the difference between η_B and $\eta_{A'}$. This is what the second part of the above proposition says. And the assumption about the size is needed for the same reason as before.

2.5 The Tipping Phenomenon

Proposition 5 *If* $1 < \tilde{T}_B < \tilde{T}_{A'}$, *the aggregate shares of currency* b *as invoicing currency in the two markets,* $\eta_{A'}$ *and* η_B, *increase at increasing rates as country B grows, i.e.* $\frac{\partial^2 \eta_{A'}}{\partial X_B^2} > 0$ *and* $\frac{\partial^2 \eta_B}{\partial X_B^2} > 0$, *when* $0 < X_B < \frac{\sqrt{\tilde{T}_{A'}\tilde{T}_B}}{1+\sqrt{\tilde{T}_{A'}\tilde{T}_B}}$.

Proposition 5 justifies the "tipping phenomenon" and its intuition is discussed below.

The assumption that $1 < \tilde{T}_B < \tilde{T}_{A'}$ states that in general, firms show stronger affinity for markets A' than for market B, and an average firm of market A' shows even stronger affinity for market A' than an average firm of market B. This is reasonable when $0 < X_B < \frac{1}{2} < \frac{\sqrt{\tilde{T}_{A'}\tilde{T}_B}}{1+\sqrt{\tilde{T}_{A'}\tilde{T}_B}}$, i.e. market B is smaller than market A, and the upper bound for X_B is larger than $1/2$.

There are two channels that contribute to the "tipping phenomenon". Both of them are from the coalescing effect. Analogous to that of an individual firm, the coalescing effect faced by the firms in the two markets can be written as $(1 - g_{A'})\eta_{A'} + g_{A'}\eta_B$ and $(1 - g_B)\eta_{A'} + g_B\eta_B$ respectively. Then $\frac{\partial^2 \eta_{A'}}{\partial X_B^2}$ and $\frac{\partial^2 \eta_B}{\partial X_B^2}$ are jointly determined by two sets of factors: one is $(\eta_B - \eta_{A'})\frac{\partial^2 g_{A'}}{\partial X_B^2}$ and $(\eta_B - \eta_{A'})\frac{\partial^2 g_B}{\partial X_B^2}$, the other is $\left(\frac{\partial \eta_B}{\partial X_B} - \frac{\partial \eta_{A'}}{\partial X_B}\right)\frac{\partial g_{A'}}{\partial X_B}$ and $\left(\frac{\partial \eta_B}{\partial X_B} - \frac{\partial \eta_{A'}}{\partial X_B}\right)\frac{\partial g_{A'}}{\partial X_B}$. The meaning of the first set is: the changes in increasing rate of the share of market B in sales of an "average" firms, i.e. $\frac{\partial^2 g_{A'}}{\partial X_B^2}$ and $\frac{\partial^2 g_B}{\partial X_B^2}$, multiplied by the given gap between η_B and $\eta_{A'}$. We can show that when $1 < \tilde{T}_B < \tilde{T}_{A'}$, $\frac{\partial^2 g_{A'}}{\partial X_B^2} > 0$ and $\frac{\partial^2 g_B}{\partial X_B^2} > 0$. Intuitively, given that the trade costs distorts the effective demands from the two markets at a constant ratio, the growth rate in effective demand share of market B is larger when X_B is larger, leading to accelerated growth of η_D. This channel is direct because the increasingly higher demand in invoicing in currency b is directly derived from the increasingly higher demand from market B.

The second channel, $\left(\frac{\partial \eta_B}{\partial X_B} - \frac{\partial \eta_{A'}}{\partial X_B}\right)\frac{\partial g_{A'}}{\partial X_B}$ and $\left(\frac{\partial \eta_B}{\partial X_B} - \frac{\partial \eta_{A'}}{\partial X_B}\right)\frac{\partial g_B}{\partial X_B}$, reflects the spillover effect of the circulation of currency b from market B to market A'. From Proposition 3, we know that $\frac{\partial \eta_B}{\partial X_B} > \frac{\partial \eta_{A'}}{\partial X_B}$, namely, when X_B rises, the gap between η_B and $\eta_{A'}$ is larger and larger. Then when X_B is larger, given one percent of increase in the share of market B in one firm's sales, its owner has to raise the share of currency b in invoicing because of a larger $(\eta_B - \eta_{A'})$, i.e. more contrasting behavior in invoicing between the two groups of competitors. As a result, the gap between $\eta_{A'}$ and η_B, like a pulling force to raise the use of currency b, becomes stronger as X_B rises. This is the indirect channel.

Since $\eta_{A'}$ and η_B are linked with each other via individual firms' choices, these two channels work on both $\eta_{A'}$ and η_B simultaneously. For example, $\eta_{A'}$ increases faster due to the spillover effect from market B to market A', then β_E decided by its seller increases faster. This leads η_B to increase faster.

The relative contribution of these two factors to the "tipping phenomenon" may vary across cases. Roughly speaking, the strength of the first channel is governed by the absolute levels of $\tilde{T}_{A'}$ and $\tilde{T}_B$ while the strength of the second is governed by the gap between $\tilde{T}_{A'}$ and $\tilde{T}_B$.[9] Consequently, the first channel is more important when $\tilde{T}_{A'}$ and $\tilde{T}_B$ are close while the second channel is more important when they are far. In addition, since $\tilde{T}_B < \tilde{T}_{A'}$, market B would be subjected to stronger saturation. Then the first channel for η_B would be weakened more quickly than $\eta_{A'}$ when X_B increases. This is why the second channel is more crucial for the convexity of η_B.

Figure 1 shows the relative importance of the two channels for η_B ($\lambda = 5$, $\Omega = 0.76$, $X_B = 0.25$ and $M^d = 0.3$). The parameter combinations within the whole range in the graph sustain the convexity. And the direct channel dominates when $\widetilde{T}_{A'}/\widetilde{T}_B$ is below the blue line (a smaller gap between $\widetilde{T}_{A'}$ and $\widetilde{T}_B$) while the indirect channel dominates when $\widetilde{T}_{A'}/\widetilde{T}_B$ is above that (a larger gap between $\widetilde{T}_{A'}$ and $\widetilde{T}_B$).

[9] $\frac{\partial \eta_{Bb}}{\partial X_B} - \frac{\partial \eta_{A'b}}{\partial X_B} \propto \frac{1}{[1-(g_B - g_{A'})]^2}\frac{\partial (g_B - g_{A'})}{\partial X_B}$ (Appendix 5.6) and $(g_B - g_{A'})$ is basically correlated with the gap between $\tilde{T}_{A'}$ and $\tilde{T}_B$ in a positive way.

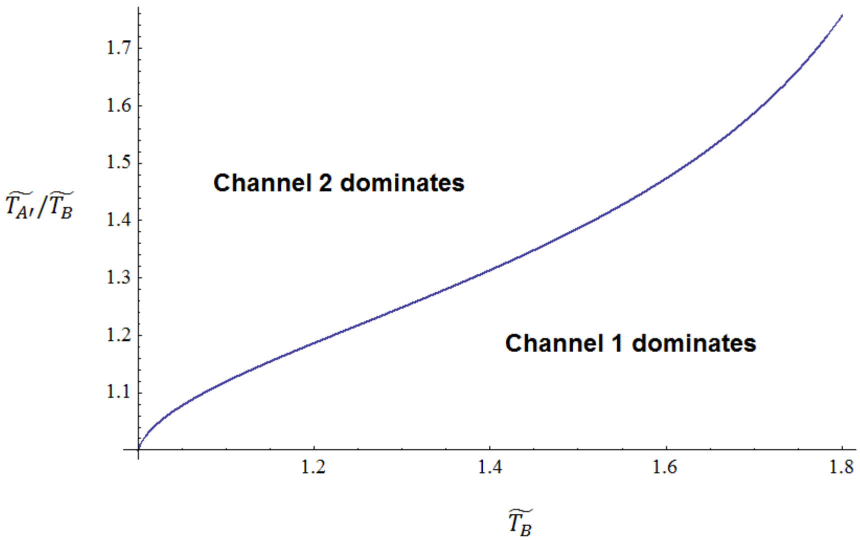

Figure 1 Relative importance of the two channels

Proposition 4 lays the foundation for the following prediction: when "tipping phenomenon" holds, currencies of large countries would occupy disproportionately larger shares of aggregate invoicing currency (see Figure 2). Consequently large countries gain excess advantages in the international statuses of their currencies.

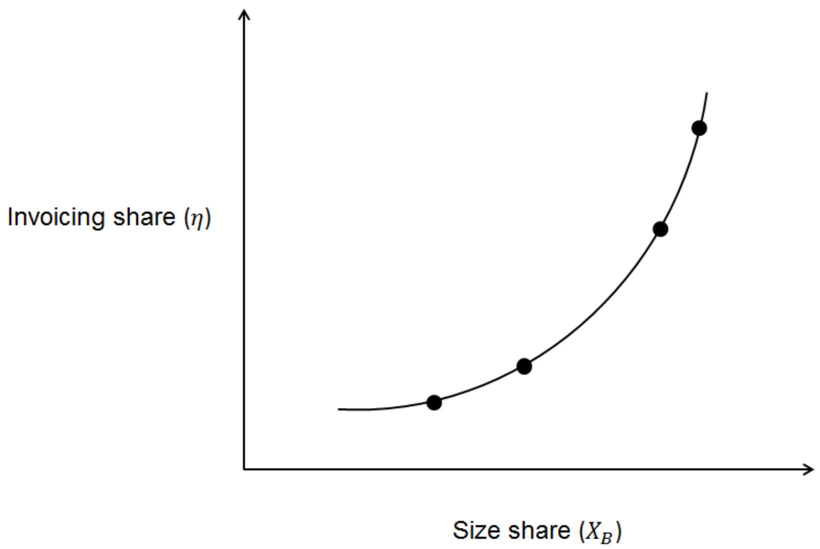

Figure 2 “Tipping phenomenon”

We next discuss the role of the trade cost $\widetilde{T}_{A'}$ and $\widetilde{T}_B$. They are essential to the convexity although they are not necessary for the internationalization itself. In other words, it affects the slope of the curve η_D as a function of X_B but does not affect the positive sign of the slope. The presence of trade costs satisfying $\widetilde{T}_{A'} > \widetilde{T}_B > 1$ distorts the curve by compressing the shares of the currency at aggregate level, with a larger degree when X_B is smaller. In addition to trade costs in traditional sense, the rationale behind this assumption comes from the network externality or economics of scale that favors the established international currencies. Countries prefer to use whatever currency others use. Then once a currency becomes a vehicle, the choice of vehicle is self-justifying. Using an analogy with language (Krugman 1984), what makes English the world’s *lingua franca* is not its simplicity or internal beauty, but its wide use. Once it is spoken among people whose native languages are not English, it is increasingly so. Similarly, an exporter chooses a currency, in the belief that other exporters are also likely to choose this one. Therefore, a list of advantages associated with this network externality, such as lower transaction costs, fully convertibility, open financial asset transaction and easily accessible financial services offered, are all conducive to the well-grounded international currencies. However, as one may already realize, to a new international currency, this type of network externality is actually a counteracting force rather than a driving force. And this is hard to avoid.[10]

This gives rise to the "tipping phenomenon" (orginally observed in official reserve holdings): "if one currency were to draw even and surpass another, the derivative of reserve currency use with respect to its determining variables would be higher in that range than in the vicinity of zero or in the range when the leading currency is unchallenged." (Chinn and Frankel 2008). By comparison, if without trade costs, the share of a currency in trade invoicing would simply increase linearly with its home country's size.

To look at a broader range of X_B, we consider the following simple and symmetric case: country A and country B are the only two big players in the world. When $0 < X_B < \frac{1}{2}$, $1 < \tilde{T}_B < \tilde{T}_{A'}$, that is, firms have more affinity to market A' in general; when $X_B = \frac{1}{2}$, $\tilde{T}_B = \tilde{T}_{A'} = 1$, firms have the same affinity to both markets and when $\frac{1}{2} < X_B < 1$, $\tilde{T}_B < \tilde{T}_{A'} < 1$, firms have more affinity to market B. Then we can obtain the following corollary:

Corollary 1 *When* $0 < X_B < \frac{1}{2}$, $\frac{\partial^2 \eta_{A'}}{\partial X_B^2} > 0$ *and* $\frac{\partial^2 \eta_B}{\partial X_B^2} > 0$. *When* $X_B = \frac{1}{2}$, $\frac{\partial^2 \eta_{A'}}{\partial X_B^2} = \frac{\partial^2 \eta_B}{\partial X_B^2} = 0$. *When* $\frac{1}{2} < X_B < 1$, $\frac{\partial^2 \eta_{A'}}{\partial X_B^2} < 0$ *and* $\frac{\partial^2 \eta_B}{\partial X_B^2} < 0$.

Interestingly, this corollary justifies the curve in Krugman (1984) (it is replicated in Figure 3). There the x-axis is the actual use of the international currency (U.S. dollar in that case) and the y-axis is the desire use of the currency. Therefore, the equilibrium point is the fixed point, the intersection point between the curve and the 45 degree line. For comparison, the curve indicated by Corollary 1 is depicted in Figure 4. Each point on this curve is already the equilibrium point corresponding to the GDP share. Though slightly different in presentation, the ideas are comparable. In that paper, the equilibrium that is attained among multiple possible candidates is implicitly determined by fundamentals while we will show later that the economic size of its home country is the most important fundamental, even the fatal one (think that dollar was dethroned finally despite the hysteresis). Krugman uses that graph to illustrate the possible collapse of dollar. He shows that if the fundamental strength of the dollar keeps deteriorating, the role of the dollar would first stand for a while, with gradual declining. But once it falls beyond some critical level, the role of the dollar would drop dramatically to a remarkably lower point and stay there even the fundamentals cease to weaken any further. Figure 4 also sheds some light on the possible sharp decline. The slope in the middle range

[10] Matsuyama et al (1993) and Helene Rey (2001) discuss natural evolutions of international currencies from international trade.

is rather steep, implying that a small variation of economic size may incur large variations in the international position of its currency. The two theories are consistent with each other and powerfully supported by the rapid switch between sterling and dollar in history.

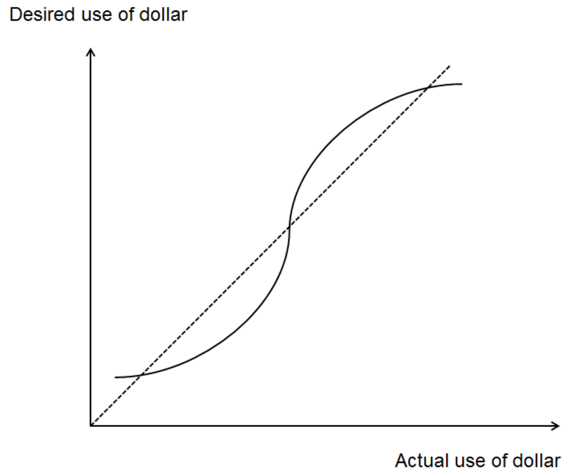

Figure 3 Krugman (1984) curve

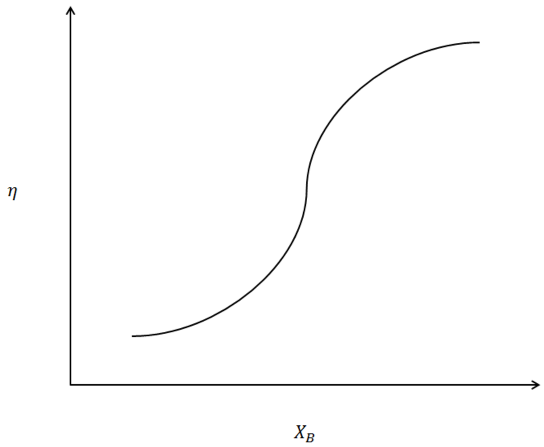

Figure 4 Corollary 1 curve

It is worthwhile to mention that the condition specified in proposition 5 is sufficient but not necessary. $\frac{\partial^2 \eta_{A'}}{\partial X_B^2}$ or $\frac{\partial^2 \eta_B}{\partial X_B^2}$ may still be positive without that. For example, we can show that $\frac{\partial^2 \eta_{A'}}{\partial X_B^2}$ is still positive when $\frac{1}{2} < X_B < \frac{2}{3}$ and $1 < \tilde{T}_{A'} < \tilde{T}_B$, or when $\frac{1}{\tilde{T}_{A'}} < \tilde{T}_B < 1 < \tilde{T}_{A'}$ and $0 < X_B < \frac{1}{2}$. However, whether they hold in more general cases, say when $\frac{1}{2} < X_B < 1$, also depends on other parameters such as Ω and M^d.

On the other hand, the exogeneity of the trade costs in our model may be a concern since they might change as X_B changes, as assumed in the symmetric case. This may be true, but the changing trends of $\widetilde{T}_{A'}$ and $\widetilde{T}_B$ are expected to be quite small in the short and middle run, considering the hysteresis effect. An international currency can keep its value if others keep using it. Thus the inertia bias favors the *status quo*. Hence, even they are indeed endogenous to economic size in the longer term, they can also be regarded as locally quasi-constant so that the degree of the convexity varies smoothly. Nevertheless, relative trade costs could change in the long run, especially when the rivalling between the fundamentals of two currencies departs substantially. We discuss the scenario in which $\widetilde{T}_{A'}$ and $\widetilde{T}_B$ are linear decreasing functions of X_B in Appendix 5.9 and show that a conclusion similar to Proposition 5 still holds with this assumption.

Finally, the dominance of a single currency can also be reinforced by "currency alliance".

There are two typical examples: one is follower countries which peg their currencies to the currency of their leader country, such as Canadian dollar and Mexican peso to U.S. dollar; the other one is monetary unions like the euro area. In such cases, the effective size is actually the combined economic size rather than the issuing country alone. Their combination would be further augmented by the coalescing effect, reducing the possible candidates for international currencies to only a few ones and driving one or two eventually to dominate.[11] This is consistent with our common sense. With this logic, a common currency issued by a monetary union would occupy a larger share of aggregate invoicing currency than the sum of the shares invoiced by its member countries in the their own currencies.[12] This explains the internationalization of euro.

2.6 Hypotheses

According to the theoretical implications, we can test the following hypotheses:

Hypothesis 1: For a sufficiently large country, firms in its small trade partner countries use its currency as invoicing currency more than its big trade partner countries.

Hypothesis 2: As a country grows relative to the rest of the world, its currency is used more as invoicing currency. Moreover, firms in its small and close trading partners adjust their invoicing currency more rapidly than those in its larger and less close trading partners.

Hypothesis 3: A currency is used as invoicing currency more in the issuing country than in the trading partners. The currency is used more in countries that trade more with the isssuing country than in countries that trade less with it. Moreover, the gap between the shares in the close and distant trading partners increases as the size of the home country grows.

Hypothesis 4: There exists a "tipping phenomenon" in the global use of a currency for invoicing purposes.

As mentioned in the introduction, "tipping phenomenon" could be observed not only over a long time horizon for a given currency, but also across currencies at a given time. For empirical tests, the observation along the time dimension for a given country imposes a higher requirement on data quantity and quality, while the observation across countries at a given time does not. So it is easier for us to test it in a "cross-sectional" way.

[11] It could be another reason for "tipping phenomenon". Although we do not explicitly explore this channel, we do consider it in our empirical work.

[12] Bacchetta and van Wincoop (2005) has a similar conclusion for the invoicing currency of a monetary union, but via a different channel.

3 The invoicing currencies in international trade

The empirical tests are conducted as follows: hypothesis 1 and 2 are tested with a data set similar to the one built by Goldberg and Tille (2008) but extended to more countries and later years. Hypothesis 3 and 4 are tested with another data set documenting multiple invoicing currencies used in Thailand's international trade. We first introduce the data sets, and then show the econometric results of the tests in parts.

3.1 Data

The data in the first set is mainly from the reports "The international role of Euro" published by European Central Bank, supplemented by reports from central banks of individual countries (U.K., Australia and Thailand)[13] and complied by us. They report the annual use of euro and dollar as invoicing currencies at country level. There are totally thirty-five countries covered. They are nine euro-area countries (Belgium, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal and Spain), thirteen European Union accession countries (Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, U.K., Slovakia and Slovenia),[14] three European Union candidate countries (Croatia, Former Yugoslav Republic of Macedonia and Turkey), one other European country (Ukraine) and nine countries outside Europe (Algeria, Australia, India, Indonesia, Pakistan, South Africa, Korea, Thailand and Tunisia). We set the earliest observations in 1998, just one year before the euro is introduced, and the latest observation is in 2010 (not balanced).[15] As far as we know, this is the most comprehensive data set on invoicing currencies across countries.

The data in the second set is from the central bank of Thailand. It includes all the invoicing currencies they use (mainly U.S. dollar, euro, British pound sterling, Japanese yen, Thai baht, and Deutsche mark) and their respective shares in trade between Thailand and fifteen European Union countries (Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and United Kingdom), three North America countries (United States, Canada and Mexico) as well as Japan in 2001-2008 (balanced). Obviously, a great virtue of this data set is that it provides the composition of multiple invoicing currencies, which allows us to test "tipping phenomenon". According to their inter-

[13] For details of this data set, please refer to the NBER working paper #11127 and Kamps (2006).

[14] Slovenia, Cyprus and Slovakia join the euro during 2007-2009. We repeat our test by adjusting for this and find the results almost unchanged.

[15] The latest observation in Goldberg and Tille (2008) is roughly in 2003.

national positions, we focus on the four major international currencies — dollar, euro, sterling and yen.

3.2 Comparison between currency invoicing in large countries and small countries

Hypothesis 1 and 2 are tested with the following general specification:

$$\begin{aligned}\beta^t_{cur,i} &= \delta_0 + \delta_1 \gamma^t_{home,i} + \delta_2 \gamma^t_{home,i} \frac{Y^t_i}{Y^t_{home}} + \delta_3 Heuro^t_i + \delta_4 EU \\ &\quad + \delta_5 year_{euro} + \delta_6 year_{euro} \frac{Y^t_i}{Y^t_{home}} + \mu^t_i\end{aligned}$$

The dependent variable $\beta^t_{cur,i}$ is the share of euro or dollar in export or import invoicing of country i in year t. Hence we have four sets of regressions: euro's share in exports, dollar's share in exports, euro's share in imports and dollar's share in imports. In general they corresponds to the β_E rather than the η_E in our model because each country is supposed to be small and take other countries' choice as given. Besides, the import invoicing currencies are also examined, which seems strange with our model for export invoicing. However, we will explain along with the results that the import invoicing can help us understand the theory from the perspective of markets.

All the explanatory variables are chosen to correspond their dependent variables. Taking the first dependent variable (euro's share in exports) as example, $\gamma^t_{home,i}$ is the share of euro area in a country's exports (here the home country of euro is euro area); $\frac{Y^t_i}{Y^t_{home}}$ is the size of the exporting country relative to the euro area; EU is an indicator variable for EU membership. $Heuro^e_t$ is the currency preference for cost hedging following Goldberg and Tille (2008). To a specific country, it is 1 when euro is a preferred hedging currency on export transactions (U.K. in our sample), 0 when euro and dollar are indifferent for hedging and -1 when dollar is a preferred hedging currency (Estonia, Hungary and Thailand in our sample). $year_{euro}$ is the number of years after the introduction of euro, taking 1998 as the base year.

The role of the first interaction term $\gamma^t_{home,i}\frac{Y^t_i}{Y^t_{home}}$ may be more interesting than one might think. According to our model, small countries are more willing to use the emerging international currency because they are more reliant on trading with this emerging market. However, this is already reflected by the first term $\gamma^t_{home,i}$. The interaction term reflects another reason pointed by our assumption: large countries have a non-negligible mass of domestic sellers who typically use their own currencies and their foreign currency followers (recall the equilibrium condition for η_B[16]). In this way, given the same reliance on trade with this particular country (the same $\gamma^t_{home,i}$), a large country uses its partner's currency less in general. An typical example is the U.K. Though both U.K. and its pound sterling have lost their preeminence for long, sterling is still used with a notable degree for its domestic and neighborhood business. By comparison, we could expect that for a small country, the weak influence of its own currency, home or abroad, would give the way to euro, even thought it has the same euro share of export/import as U.K. By the way, the second interaction term $year_{euro}\frac{Y^t_i}{Y^t_{home}}$ contains the both reasons for the contrast between large countries and small countries.

The role of the the time trend variable $year_{euro}$ also needs discussion. According to hypothesis 2, the internationalization of a currency is promoted by the growth of its home country. In euro's case, it is captured by this time trend variable because the euro area's GDP share X_{euro}, exploding immediately at the introduction of euro, is further enlarged gradually afterwards, as more and more EU countries join the the euro (Slovenia from 2007, Cyprus and Malta from 2008, and Slovakia from 2009, etc.). Besides, this time variable also contains the inertia in adjustments by countries. The interaction term $year_{euro}\frac{Y^t_i}{Y^t_{home}}$ catches the difference in adjustment paces of small and large countries.

Table 1 presents the results from the specification for the euro invoicing in exports. Explanatory variables are introduced and accumulated one by one.[17] As expected, the share of euro area in country exports is strongly positive for the use of euro and contributes more than 60% to the explanatory power to account for the cross-country and intertemporal variation of euro's share in export invoicing. This corroborates the necessity to distinguish countries according to their trade relationship with the home country.

The negative and significant coefficient estimates before the second explanatory variable shows that euro is less used in exports by countries that are relatively large, with the share of exports to euro area controlled. This result is quite similar to those of Goldberg and Tille (2008), both in quality and quantity. The magnitude of this coefficient drops nearly by half when the second interaction term is introduced (last column) since these two terms are partially correlated.

EU members use euro more widely, suggested by column (3). This is natural and also an evidence for hypothesis 3. However, the hedging motive seems not important in our results.

[16] In this sense, rather than β_{EB} only, the dependent variables also include the component of η_{Eb}.

[17] To examine the vehicle roles of euro and dollar, we eliminate euro area countries for euro invoicing analyses and include them for dollar invoicing analyses.

This validates our simplification that cost hedging term is approximately zero in the model.

The time trend $year_{euro}$ enters with significant and positive coefficient, supporting hypothesis 2. In general, the use of euro increases over time. What is more, the increase is contributed by smaller countries with a relatively larger degree, signified by strongly negative coefficient for the interaction term $year_{euro}\frac{Y_i^t}{Y_{home}^t}$. If one takes a further inspection into the data, she will find that relatively large countries, such as U.K., Australia and Denmark, are indeed slower adjustors.

The last column incorporates transaction cost into the regression. The transaction cost is calculated in a way that is standard in literature[18] — the median difference in pips on using euros versus pips on using dollars for each year. Consistent with Goldberg and Tille (2008), it is insignificant to explain invoicing currency choice.[19]

Table 2 presents the results for dollar invoicing in exports. The features showed by the first two coefficients are similar to euro. But the magnitude of the coefficients before the second term are much larger. This is because we use the GDP of each member country in EU rather than the GDP of the whole EU. If we replace it with the GDP of the whole EU, the coefficient will fall to roughly -2.5 but still show significance at 1% level (not reported). The features showed by the last four coefficients are almost opposite to the euro case, which are reasonable as euro and dollar are competitors, at least in Europe. The negative coefficient before the time trend supports what we predict for monetary union in section 2: even though euro gains popularity mainly at the expense of the original currencies of its member countries, it crowds out the dollar in Europe a little bit. And the last term shows that larger countries are obviously slow responders to this change.

Table 3 and 4 report the results from euro and dollar invoicing in imports. We can interpret the results from the perspective of markets. Intuitively, suppose we decompose $\eta_{A'}$ into η_A and η_C, we could expect that in equilibrium $\eta_A < \eta_C < \eta_B$. The logic is similar to the interpretation for the interaction term $\gamma_{home,i}^t\frac{Y_i^t}{Y_{home}^t}$: small countries have fewer currency followers home or abroad. And with less mass as ballasts, import invoicing currency of small countries will also be affected more easily. Thus we expect the import invoicing currencies to share similar patterns with those of export invoicing currencies.

[18] Goldberg and Tille (2008) and Chen Hongyi et al (2009)

[19] Since some missing observations in data for transaction costs limit the total observations we can use, we report the results without it from now on. However, when we repeat our analysis with it, the results are almost unaltered, as in column (7).

Generally speaking, they indeed do. Nonetheless, the time trend term $year_{euro}$ is not significant in the case of dollar in import invoicing. This implies that a world-wide "average" exporter does not lessen the use of dollar as invoicing currency. In other words, euro is more a regionally international currency than a globally internationally currency. But the pace difference still stands for both euro and dollar. Further, if we restrict our sample to EU member countries in the last case, the desired patterns are much stronger (column 7 in Table 4). This, again, reminds us that the variations of trade relationship or different extents of economic integration among countries should not be ignored to understand the breadth and depth of internationalization of a currency.

3.3 Comparison across invoicing currencies

For comparison across multiple invoicing currencies, the second data set has an obvious limitation — its narrow coverage (only Thailand's trade). But we can turn this disadvantage to an advantage when testing hypothesis 3. With one partner of the trade being controlled, we obtain a check on the different influences of an international currency in different geographic areas, and verify hypothesis 3 straightforwardly. The data reports the aggregate compositions of invoicing currencies between Thailand and three trade unions: EU 15, NAFTA, and ASEAN. The shares of U.S. dollar, euro, British pound sterling and Japanese yen in Thailand's export invoicing to these three unions and their home countries' global GDP shares are scattered in Figure 5.

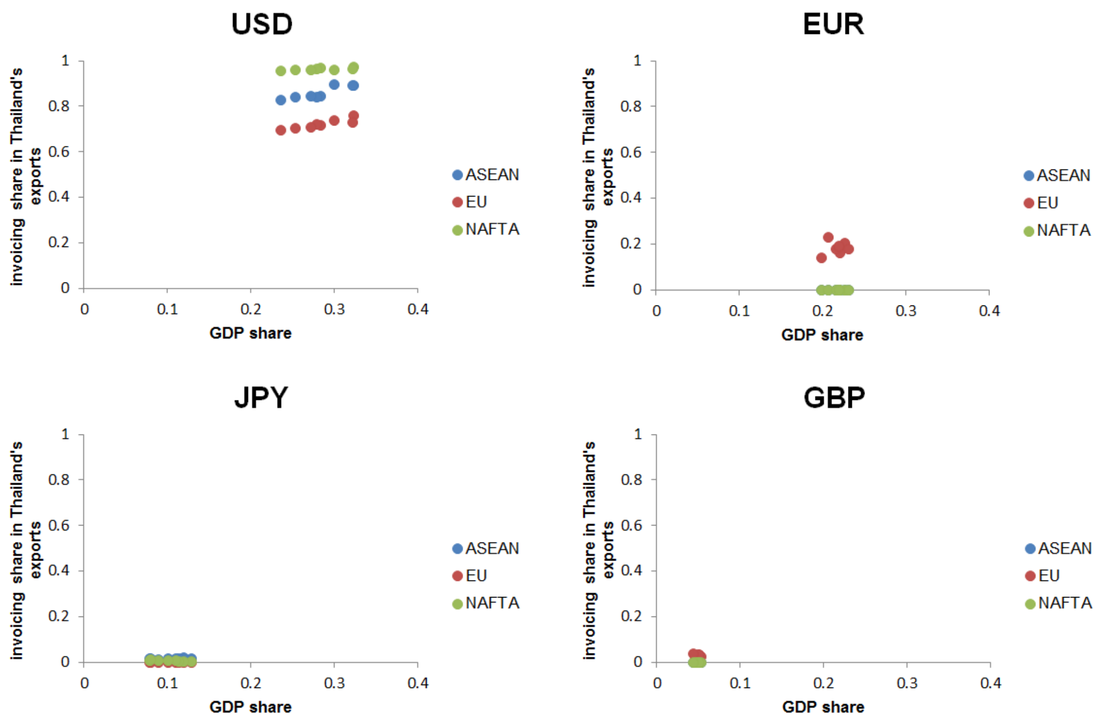

Figure 5 Four international currencies: export invoicing share to ASEAN/EU/NAFTA versus its home country's GDP share

It is clear that regardless of the destination, dollar is the absolute number one invoicing currency, while euro is the number two, ahead of yen and pound sterling. Beyond that, two patterns are noteworthy: first, an international currency gains more acceptance in their home and neighborhood markets, namely, U.S. dollar to NAFTA, euro and sterling to EU, and Japanese yen to ASEAN, respectively. Second, the larger is the home country, the larger is the gap. The four subfigures are ranked at a descending order of its home country's size. Obviously, the gap associated with dollar is the largest, followed by that associated with euro, and the gaps corresponding to yen and sterling are much smaller.[20] These characteristics are also significant in Thailand's import invoicing (see Figure 7 in Appendix).

[20] If one regress the invoicing shares of each currency on the dummy of its home, the rank of the coefficient estimates (all positive) is also consistent with the rank of GDP share except for Japanese yen. This is probably because Japan adopts a pricing-to-market policy which encourages exporters to invoice in local currency of foreign markets (Takatoshi et al 2010). From another perspective, this policy affirms the mechanism of the coalescing effect.

3.4 "Tipping phenomenon"

Chinn and Frankel (2007) document a strong non-linear relationship between currency shares in official reserves and GDP shares. A similar "tipping phenomenon" is expected to be observed in invoicing currencies. However, there is rarely any econometric study on this, since the data on trade invoicing-currency is much less comprehensively quantifiable than that on reserve currency holdings.

Figure 6 displays the shares of the four currencies in trade invoicing to EU 15 countries in 2001-2008. The existence of "tipping phenomenon" is apparent from inspection of the diagram.

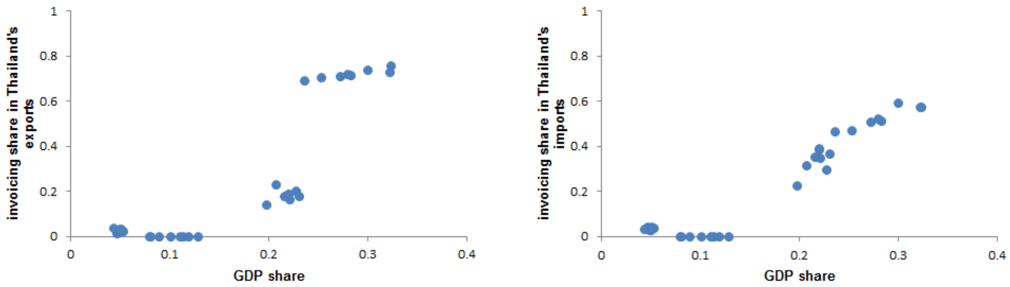

Figure 6 Invoicing currency share versus GDP share (left: export; right: import)

To examine the "tipping phenomenon", we compare the fitness between the following two specifications:

$$\begin{aligned}\beta^t_{cur} &= \delta_1 x^t_{issuer} + \delta_2 \gamma^t_{partner,issuer} + \delta_3 \gamma^t_{Thailand,issuer} + \delta_4 (bid - ask)^t_{cur,baht} \\ &\quad + \delta_5 regime + \varepsilon^t_{cur}\end{aligned}$$

$$\begin{aligned}\log \frac{\beta^t_{cur}}{1-\beta^t_{cur}} &= \delta_1 x^t_{issuer} + \delta_2 \gamma^t_{partner,issuer} + \delta_3 \gamma^t_{Thailand,issuer} + \delta_4 (bid - ask)^t_{cur,baht} \\ &\quad + \delta_5 regime + \nu^t_{cur}\end{aligned}$$

The two specifications only differ in the form of the dependent variable. The one in the second is the logit transformation of the one is the first. The logit function $f(x) = \log \frac{x}{1-x}$ is concave when $0 < x < \frac{1}{2}$ and convex when $\frac{1}{2} < x < 1$. Therefore the non-linearity embedded in the "tipping phenomenon" as well as the Krugman (1984) curve can be offset by this transformation, leading to better linear fitness. So the improvement in the linear fitness when moving from the first specification to the second is an indicator for the strength of the non-linearity.

Among the independent variables, x^t_{issuer} is the GDP share of the issuing country of one particular international currency in year t. The variables $\gamma^t_{partner,issuer}$ and $\gamma^t_{Thailand,issuer}$ are the shares of exports (imports) to the issuing country in the total exports (imports) of that partner country and Thailand, respectively. If the partner country happens to be the issuing country, we set $\gamma^t_{partner,issuer} = 1$. The variable $pip(cur - baht)^t$ is the transaction costs between Thai Baht and the currency in question. The variable *regime* is the extent to which the exchange rate of the currency of the partner country fluctuates with the exchange rate of that international currency, with respect to the same base currency.

As mentioned in the section 2, the exchange rate regime could be another determinant for invoicing currency decision, so we include it into our regression. The more an international currency weighs in a country's currency basket, the more preferable it is to the exporters from this country. To measure the relative weight of major international currencies in one country's currency basket, we adopt the method introduced by Frankel and Wei (1994) and applied by other studies.[21] More specifically, we use the daily exchange rate data from Bloomberg (last price)[22] and run the following regression

$$\begin{aligned}\Delta\log(S_{partnercurrency/SwissFranc}) &= \kappa_1\Delta\log(S_{USD/SwissFranc}) + \kappa_2\Delta\log(S_{EUR/SwissFranc})\\ &\quad +\kappa_3\Delta\log(S_{GBP/SwissFranc}) + \kappa_4\Delta\log(S_{JPY/SwissFranc})\\ &\quad +\kappa_0 + \tau^t_{partnercurrency}\end{aligned}$$

The fluctuations of exchange rates of four major international currencies are utilized to explore the fluctuations of the exchange rate of a national currency, by taking Swiss Franc as the base currency. The coefficients suggest the weight of one international currency in this country's currency basket.[23] Our estimates show that in North America, Canadian dollar partially comove with U.S. dollar while Mexican peso is almost completely pegged to U.S. dollar. U.K., Denmark and Sweden are the non-euro countries among the fifteen EU countries. Danish krona is closely pegged to euro while Swedish krona comove with euro with a very low degree.

Unlike studies that attempt to ascertain the determinants of the international role of a currency, we do not include a lagged variable such as β^{t-1}_{cur}, because this variable is highly

[21] e.g. Eichengreen (2006) and Frankel (2007)

[22] The period is Jan.1st 2001 to Dec. 31st 2008. We estimate the coefficients on a year basis.

[23] but not equal to that weight exactly. See Frankel and Wei (1994).

correlated with the x^t_{home}, as one can expect. What we want to capture is a long run trend of the relationship between the vehicle currency role and its home country's economic size.

Table 5 and 6 present the comparisons before and after logit transformation in pairs. Quite clearly, for each pair with exactly the same sample, judged by the adjusted R^2, the specifications after logit transformation are systematically more successful to explain the shares of international currencies in invoicing. In fact, considering the automatic elimination of the observations of zero values by the transformation, our estimations about the "tipping phenomenon" are actually downward biased since these observations can sharpen the contrast across currencies.

The economic size measured by GDP share accounts for more than 50% of differences in the invoicing currency shares, showed by column (1) and (2). This establishes the absolute dominance of economic size among a number of economic fundamentals (current account status, inflation rate as well as appreciation/depreciation and so forth). Besides, the additional explanatory power with logit transformation is mainly contributed by an obvious rise in the significance of this term, which precisely indicates the non-linearity. This echoes a similar finding in Chen Hongyi et al (2009). They demonstrate that the global GDP share is remarkably more important than other fundamental variables to account for the currency shares of reserve holdings.

As expected, the share of the international-currency-issuing country in total export/import of one country $\gamma^t_{partner,home}$ enters with positive signs. However, it may be puzzling that the share of the issuing country in total import of Thailand $\gamma^t_{Thailand,home}$ enters with negative signs. This is because the descending order of trade closeness of Thailand's importing partners is Japan, EU 15, U.S. and U.K. which happens to be negatively correlated with their economic sizes. On the other hand, we can interpret the strength and the robustness of the role of economic sizes from this result, as the country-specific trade relationships are not able to distort the positions of international currencies, even locally.

Another two interesting points are with respect to the effects of transaction costs and exchange rate regime. Unlike the previous tests, the transaction cost is significant in this test. Trade partner involved here do concern about the transaction costs. A possible reason for that is it could be easier and less expensive for exporters from European countries (the majority in the first sample) to switch between euro and dollar than for Thai firms to convert baht to dollar, euro, sterling or yen.

The negative signs of the coefficients before the exchange regime term also seem unexpected. This is because the majority of the sampled countries are European countries which do not peg to U.S. dollar but to euro. This, just as the export/import share discussed above, indirectly corroborates the notable dominance of economic size in determining the international status of a currency. If we control for the currency type, this coefficient will become positive (not reported).

Finally, the "tipping phenomenon" can also be tested in specification with quadratic form or cubic form (see Table 7). With the quadratic form, the second order term is strongly positive, supporting the non-linearity. With the cubic form, the third order term is also significant, with a negative sign. This implies that when GDP share is relative smaller (smaller than 0.20 in this case), the non-linearity is convexity while when the GDP share is relative larger (larger than 0.20), the non-linearity turns to concavity. In terms of the number of statistically significant coefficients, the cubic form fits better. Thus it indicates a possibility that although the economic power of U.S. does not completely dominate (its global GDP share is about 20%-30%), dollar is already positioned on the second half of the Krugman curve.

4 Conclusion

We develop a theory to explain the emergence of a currency as a vehicle currency for trade settlement as the issuing country's economic size grows relative to the rest of the world. The fundamental mechanism for the dynamics of the emergence of the currency is the coalescing effect: exporters have strong incentive to use the currency mix that their competitors use to invoice their exports. When a country's market expands, exporters to that market tend to use a larger share of the local currency to invoice its exports. Since exporters would require the price levels across markets to be not so different as to allow goods-arbitrage to occur, the use of the currency in its home and nearby markets would spread to other markets. The positive feedback between the use of a currency as a vehicle currency by an individual firm and that by its competitors helps to accelerate the use of that currency for trade settlement.

Furthermore, the propagation would start from its smaller and closer trading partners to larger and less close trading partners. This is because the smaller and closer partners often have higher trade/GDP ratio with this rapidly growing country and the smaller countries' currencies are less likely to be used by domestic and foreign firms as invoicing currencies. On the contrary, the larger and less close partners have lower trade/GDP ratios with the growing country, and the larger countries' currencies are more likely to be used by domestic and foreign firms as vehicle currencies. Consequently, the emerging currency would gain acceptance from countries that rely more on trade with the emerging country before the acceptance is spread to countries that rely less on trade with the emerging country.

More importantly, we derive a non-linear but positive relationship between the share of global use of a currency as an invoicing currency and the economic size of the issuing country. We show that the relationship is convex when the economic size of the issuing country is small relative to the rest of the world and concave when it is relatively large. This confirms Krugman's (1984) curve. The convex part corresponds to the tipping phenomenon in the sense that currencies of larger countries have disproportionately higher share of global use as vehicle currency. This non-linearity is generated by the smaller transaction cost associated with the established international currency compared with that associated with the currency of the emerging economy. With these frictions favoring the established international currency, the global use of the currency of an emerging country increases very slowly in the beginning, even as the country grows very fast. However, this rate of increase accelerates as the country's relative size continues to grow. When the economic size of the country reaches a certain tipping point, the global use of its currency can abruptly increase, and it becomes a widely used vehicle currency in a relatively short period of time. When the currency gains sufficient dominance as a vehicle currency, the rate of increase of its global use decelerates even as the relative economic size of the issuing country continues to grow. This corresponds to the concave part of the curve.

Although our model makes use of many simplifying assumptions, it is sufficient to capture the basic insights and intuition behind the mechanism at work. Moreover, the hypotheses generated by the theory are testable, and tested, using the scarce and scattered data on invoicing currencies in a number of countries. The empirical results are consistent with the implications of the model. The results also strongly indicate the significance of (i) the share of global trade of the issuing country as well as (ii) its relative economic size, in determining the share of global use of the currency for trade settlement. These channels are also predicted to be of great significance by our theory.

Our theory and the empirical evidence show that one should expect the pace of increase in the global use of the Chinese RMB for trade settlement to be slow in the near future, even as China grows fast relative to the rest of the world. This is so especially because China's financial system is quite immature and underdeveloped, and the RMB is not fully convertible. Besides tackling these shortcomings of its financial system, China can speed up internationalization of the RMB by facilitating the coalescing effect to take place, as well as facilitating its major trading partners (i.e. the smaller or closer countries) to use RMB for trade settlement by making it less costly for these foreign firms to buy and sell RMB. Some of these measures have already been undertaken in recent years.[24]

[24] For instance, the central bank of China has signed currency swap contracts with central banks of twenty countries. The countries are mainly in Asia, including South Korea and Japan.

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5 Appendix

5.1 Solving for the invoicing currency of a firm from country C: β_C

We maximize the profits by choosing P_C^k and β_C in separate steps.

Step 1: choose P_C^k to maximize the expected profit at steady state by taking β_C as given:

$$\begin{aligned}
& (\lambda-1)(P_C^k)^{1-\lambda} E\left[D_C S_{ckC}^{1-\lambda}(T_{CA'}^{-\lambda} S_{ca}^{\lambda} P_{A'}^{\lambda} C_{A'} + T_{CB}^{-\lambda} S_{cb}^{\lambda} P_B^{\lambda} C_B)\right] \\
= \ & \lambda \alpha^{\frac{1}{\alpha}-1}(P_C^k)^{-\frac{\lambda}{\alpha}} E\left[D_C W_C S_{ckC}^{-\frac{\lambda}{\alpha}}(T_{CA}^{-\lambda} S_{ca}^{\lambda} P_{A'}^{\lambda} C_{A'} + T_{CB}^{-\lambda} S_{cb}^{\lambda} P_B^{\lambda} C_B)^{\frac{1}{\alpha}}\right]
\end{aligned}$$

Step 2: Linearize the profit function around its steady state

$$\Pi_C = \Pi_{Css}(1+\pi_C)$$

where $\pi_C = \frac{\Pi_C - \Pi_{Css}}{\Pi_{Css}}$.

Since the profit at the steady state is a constant independent of the invoicing currency, we only need to maximize π_C. And π_C can be expressed as:

$$\begin{aligned}
\pi_C \ = \ & \frac{\lambda}{\lambda-\alpha(\lambda-1)} E\left\{D_C S_{ckC} P_C^k \left[\widehat{\left(\frac{S_{ckC} T_{CA'} P_{Ck}}{S_{ca} P_{A'}}\right)^{-\lambda}} C_{A'} + \left(\frac{S_{ckC} T_{CB} P_C^k}{S_{cb} P_B}\right)^{-\lambda} C_B\right]\right\} \\
& -\frac{\alpha(\lambda-1)}{\lambda-\alpha(\lambda-1)} E\left\{D_C W_C \alpha^{\frac{1}{\alpha}} \left[\widehat{\left(\frac{S_{ckC} T_{CA'} P_C^k}{S_{ca} P_{A'}}\right)^{-\lambda}} C_{A'} + \left(\frac{S_{ckC} T_{CB} P_C^k}{S_{cb} P_B}\right)^{-\lambda} C_B\right]^{\frac{1}{\alpha}}\right\}
\end{aligned}$$

Log approximate it to second order,

$$\begin{aligned}
\pi_C \ = \ & \frac{\lambda}{\lambda-\alpha(\lambda-1)}\Big[(1-\lambda)P_C^k + Ed_C + (1-\lambda)Es_{ckC} + E\phi_C + \frac{1}{2}E[d_C + (1-\lambda)s_{ckC} \\
& +\phi_C]^2\Big] - \frac{\alpha(\lambda-1)}{\lambda-\alpha(\lambda-1)}\Big[-\frac{\lambda}{\alpha}P_C^k + Ed_C + Ew_C - \frac{\lambda}{\alpha}Es_{ckC} + \frac{1}{\alpha}E\phi_C \\
& +\frac{1}{2}E(d_C + w_C - \frac{\lambda}{\alpha}s_{ckC} + \frac{1}{\alpha}\phi_C)^2\Big]
\end{aligned}$$

where

$$\phi_C = (1-\gamma_C)(\lambda s_{ca} + \lambda p_{A'} + c_{A'}) + \gamma_C(\lambda s_{cb} + \lambda p_B + c_B)$$

where $\gamma_C = \frac{T_{CB}^{-\lambda} S_{cb}^{\lambda} P_B^{\lambda} C_B}{T_{CA'}^{-\lambda} S_{ca}^{\lambda} P_{A'}^{\lambda} C_{A'} + T_{CB}^{-\lambda} S_{cb}^{\lambda} P_B^{\lambda} C_B}$ is the share of sales in market B of a firm from country C at the steady state.

Take the derivative of π_C with respect to β_C

$$\begin{aligned}\frac{\partial \pi_C}{\partial \beta_C} &= \frac{\lambda}{\lambda-\alpha(\lambda-1)}\left[(1-\lambda)^2 E s_{ck_C}\frac{\partial s_{ck_C}}{\partial \beta_C}+(1-\lambda)E(d_C+\phi_C)\frac{\partial s_{ck_C}}{\partial \beta_C}\right] \\ &\quad -\frac{\alpha(\lambda-1)}{\lambda-\alpha(\lambda-1)}\left[(\frac{\lambda}{\alpha})^2 E s_{ck_C}\frac{\partial s_{ck_C}}{\partial \beta_C}-\frac{\lambda}{\alpha}E[d_C+w_C+\frac{1}{\alpha}\phi_C]\frac{\partial s_{ck_C}}{\partial \beta_C}\right]\end{aligned}$$

Since $s_{ck_C}=(1-\beta_C)s_{ca}+\beta_C s_{cb}=s_{ca}+\beta_C s_{ab}$, then $\frac{\partial s_{ck}}{\partial \beta_C}=s_{ab}$. Substituting it as well as $p_{A'}=(1-\eta_{A'})s_{aa}+\eta_{A'}s_{ab}=\eta_{A'}s_{ab}$ and $p_B=\eta_B s_{bb}+(1-\eta_B)s_{ba}=(1-\eta_B)s_{ba}=-(1-\eta_B)s_{ab}$ into the above expression and setting $\frac{\partial \pi_C}{\partial \beta_C}=0$, we obtain:

$$\begin{aligned}\beta_C &= \Omega[(1-\gamma_C)\eta_{A'}+\gamma_C\eta_B]+(1-\Omega)\frac{E(m_C s_{ab})}{E(s_{ab})^2} \\ &= \Omega[(1-\gamma_C)\eta_{A'}+\gamma_C\eta_B]+(1-\Omega)\rho_C\end{aligned}$$

where $\Omega=\frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+\alpha}$, $m_C=\frac{1-\alpha}{\alpha}[(1-\gamma_C)c_{A'}+\gamma_C c_B]+w_C$ and $\rho_C=\frac{E(m_C s_{ab})}{E(s_{ab})^2}+\frac{E(s_{ac}s_{ab})}{E(s_{ab})^2}$.

5.2 Proof to Proposition 1

From Appendix 5.1, $\gamma_E=\frac{T_{EB}^{-\lambda}S_{ab}^{\lambda}P_B^{\lambda}C_B}{T_{EA'}^{-\lambda}P_{A'}^{\lambda}C_{A'}+T_{EB}^{-\lambda}S_{ab}^{\lambda}P_B^{\lambda}C_B}=\frac{S_{ab}^{\lambda}P_B^{\lambda}C_B}{T_E P_{A'}^{\lambda}C_{A'}+S_{ab}^{\lambda}P_B^{\lambda}C_B}$. Given $T_A>T_C$, $\gamma_A<\gamma_C$. And since $\rho_A\approx 0$ and $\rho_C\approx 0$, then $\beta_A<\beta_C$.

5.3 Proof to Proposition 2

Given $X_B=\frac{S_{ab}^{\lambda}P_B^{\lambda}C_B}{P_{A'}^{\lambda}C_{A'}+S_{ab}^{\lambda}P_B^{\lambda}C_B}$, $\gamma_E=\frac{X_B}{T_E+(1-T_E)X_B}$ and $\frac{\partial\gamma_E}{\partial X_B}=\frac{T_E}{[T_E+(1-T_E)X_B]^2}$. Obviously, γ_E increases in X_B. Given $\eta_{A'}$ and η_B fixed, $\frac{\partial\beta_{Eb}}{\partial X_B}=(\eta_B-\eta_{A'})\frac{\partial\gamma_E}{\partial X_B}$. It is easy to verify that given $T_A>T_C$, $\frac{\partial\beta_{AB}}{\partial X_B}<\frac{\partial\beta_{CB}}{\partial X_B}$ if and only if $X_B<\frac{\sqrt{T_A T_C}}{1+\sqrt{T_A T_C}}$.

5.4 Solving for the aggregate invoicing currency: $\eta_{A'}$ and η_B

Market A' clears when

$$N_A Y_{AA'}+N_B Y_{BA'}+N_C Y_{CA'}+N_A^d Y_{A'A'}^d=C_{A'}$$

where N_E is the number of exporting firms in country E . Substituting the demand functions into it, we obtain

$$\begin{aligned}&N_A\left(\frac{S_{ak_A}T_{AA'}P_{AA'}}{S_{aa}P_{A'}}\right)^{-\lambda}C_{A'}+N_B\left(\frac{S_{bk_B}T_{BA'}P_{BA'}}{S_{ba}P_{A'}}\right)^{-\lambda}C_{A'}+N_C\left(\frac{S_{ck_C}T_{CA'}P_{CA'}}{S_{ca}P_{A'}}\right)^{-\lambda}C_{A'} \\ &+N_A^d\left(\frac{P_{A'A'}^d}{P_{A'}}\right)^{-\lambda}C_{A'}=C_{A'}\end{aligned}$$

Log linearize the both sides of the above equation and make use of $\log\left(\frac{S_{Ek_E}}{S_{ea}}\right) = (1-\beta_{Xb})s_{aa} + \beta_{Xb}s_{ab} = \beta_{Xb}s_{ab}$ and $p_{A'} = (1-\eta_{A'})s_{aa} + \eta_{A'}s_{ab} = \eta_{A'}s_{ab}$

$$M_{AA'}\beta_A + M_{BA'}\beta_B + M_{CA'}\beta_C = \eta_{A'}$$

where $M_{EA} = N_E\left(\frac{S_{ek_E}T_{EA'}P_{EA'}}{S_{ea}P_{A'}}\right)^{-\lambda}$ is the market share of firms from country E in market A'.

The equilibrium condition for market B can be derived similarly. Then by substituting all βs into the two equilibrium conditions showed below, the aggregate invoicing currency $\eta_{A'}$ and η_B can be solved.

$$\eta_{A'} = M_{AA'}\beta_A + M_{BA'}\beta_B + M_{CA'}\beta_C = f_{A'}\eta_{A'} + g_{A'}\eta_B + h_1 \quad (2)$$

$$\eta_B = M_{AB}\beta_A + M_{BB}\beta_B + M_{CB}\beta_C + M_B^d \cdot 1 = f_B\eta_{A'} + g_B\eta_B + h_2 \quad (3)$$

where $f_{A'}$, $g_{A'}$, $h_{A'}$, f_B, g_B and h_B are specified in (1).

5.5 Proof to Proposition 3

From Appendix 5.4,

$$\eta_{A'} = \frac{(1-g_B)h_{A'} + g_{A'}h_B}{1 - f_{A'} - g_B - f_Bg_{A'} + f_{A'}g_B} \quad (4)$$

$$\eta_B = \frac{f_Bh_{A'} + (1-f_{A'})h_B}{1 - f_{A'} - g_B - f_Bg_{A'} + f_{A'}g_B} \quad (5)$$

Then we can get the following results easily:

(1) The common denominator $1 - f_{A'} - g_B - f_Bg_{A'} + f_{A'}g_B = [1-\Omega(1-M_A^d)](1-g_B) + [1-\Omega(1-M_B^d)]g_{A'} = [1-\Omega(1-M^d)](1+g_{A'}-g_B) > 0$;

(2) $f_Bh_{A'} + h_B(1-f_{A'}) > 0$ because $h_B > h_{A'} > 0$, which, together with result (1), in turn implies that $\eta_{A'} > 0$;

(3)$[f_Bh_{A'} + h_B(1-f_{A'})] - [(1-g_B)h_{A'} + g_{A'}h_B] = [1-\Omega(1-M_A^d)]h_B - [1-\Omega(1-M_B^d)]h_{A'} = [1-\Omega(1-M^d)](h_B - h_{A'}) > 0$, which together with result (1), leads to $\eta_B > \eta_{A'}$[25].

5.6 Proof to Proposition 4

From (2) and (3)

$$\frac{\partial\eta_{A'}}{\partial X_B} = f_{A'}\frac{\partial\eta_{A'}}{\partial X_B} + g_{A'}\frac{\partial\eta_B}{\partial X_B} + \left(\eta_{A'}\frac{\partial f_{A'}}{\partial X_B} + \eta_B\frac{\partial g_{A'}}{\partial X_B} + \frac{\partial h_{A'}}{\partial X_B}\right) \quad (6)$$

$$\frac{\partial\eta_B}{\partial X_B} = f_B\frac{\partial\eta_{A'}}{\partial X_B} + g_B\frac{\partial\eta_B}{\partial X_B} + \left(\eta_{A'}\frac{\partial f_B}{\partial X_B} + \eta_B\frac{\partial g_B}{\partial X_B} + \frac{\partial h_B}{\partial X_B}\right) \quad (7)$$

[25] In fact,we also need $M^d < \overline{M^d}$ at which $\eta_B = 1$.

In each equation, the first two terms on the right hand side come from the increase of the aggregate shares of currency b invoiced by a firm's competitors, the middle two terms come from the increase of average shares of sales of firms and the last one come from the changes in the macroeconomic fluctuations. Since ρ_E and M^d are assumed exogenous, then $\frac{\partial h_{A'}}{\partial X_B} = \frac{\partial h_B}{\partial X_B} = 0$.

Note that the above equation system can also be expressed in the following form:

$$\frac{\partial \eta_{A'}}{\partial X_B} = f_{A'}\frac{\partial \eta_{A'}}{\partial X_B} + g_{A'}\frac{\partial \eta_B}{\partial X_B} + i_{A'} \tag{8}$$

$$\frac{\partial \eta_B}{\partial X_B} = f_B\frac{\partial \eta_{A'}}{\partial X_B} + g_B\frac{\partial \eta_B}{\partial X_B} + i_B \tag{9}$$

where $f_{A'}$, $f_{A'}$, g_B and g_B are the same as before and $i_{A'}$ and i_B are

$$i_{A'} = \left(\eta_{A'}\frac{\partial f_{A'}}{\partial X_B} + \eta_B\frac{\partial g_{A'}}{\partial X_B} + \frac{\partial h_{A'}}{\partial X_B}\right) = (\eta_B - \eta_{A'})\frac{\partial g_{A'}}{\partial X_B}$$

$$i_B = \left(\eta_{A'}\frac{\partial f_B}{\partial X_B} + \eta_B\frac{\partial g_B}{\partial X_B} + \frac{\partial h_B}{\partial X_B}\right) = (\eta_B - \eta_{A'})\frac{\partial g_B}{\partial X_B}$$

The second equalities of above equations are derived based on $f_{A'} = \Omega(1 - M^d) - g_{A'}$ and $f_B = \Omega(1 - M^d) - g_B$ respectively. Then $\frac{\partial \eta_{A'}}{\partial X_B}$ and $\frac{\partial \eta_B}{\partial X_B}$ can be solved. $\frac{\partial \gamma_A}{\partial X_B} > 0$, $\frac{\partial \gamma_B}{\partial X_B} > 0$ and $\frac{\partial \gamma_C}{\partial X_B} > 0$ imply $\frac{\partial g_{A'}}{\partial X_B} > 0$ and $\frac{\partial g_B}{\partial X_B} > 0$. Together with $(\eta_B - \eta_{A'}) > 0$ from Appendix 5.5, we know $i_{A'} > 0$ and $i_B > 0$. Using the same logic in Appendix 5.5,

$$(1 - g_B)i_{A'} + g_{A'}i_B > 0 \quad \Rightarrow \quad \frac{\partial \eta_{A'}}{\partial X_B} > 0$$

$$f_B i_{A'} + (1 - f_{A'})i_B > 0 \quad \Rightarrow \quad \frac{\partial \eta_B}{\partial X_B} > 0$$

From Appendix 5.5, we also know

$$\frac{\partial \eta_{A'}}{\partial X_B} < \frac{\partial \eta_B}{\partial X_B} \Leftrightarrow i_{A'} < i_B \Leftrightarrow \frac{\partial g_{A'}}{\partial X_B} < \frac{\partial g_B}{\partial X_B}$$

Invoking $g_{A'} = \frac{X_B}{\tilde{T}_{A'} + (1 - \tilde{T}_{A'})X_B}$ and $g_B = \frac{X_B}{\tilde{T}_B + (1 - \tilde{T}_B)X_B}$, we obtain that $\frac{\partial g_{A'}}{\partial C_B} < \frac{\partial g_B}{\partial C_B}$ when $0 < X_B < \frac{\sqrt{\tilde{T}_{A'}\tilde{T}_B}}{1 + \sqrt{\tilde{T}_{A'}\tilde{T}_B}}$. Therefore, $\frac{\partial \eta_{A'}}{\partial X_B} < \frac{\partial \eta_B}{\partial X_B}$ when $0 < X_B < \frac{\sqrt{\tilde{T}_{A'}\tilde{T}_B}}{1 + \sqrt{\tilde{T}_{A'}\tilde{T}_B}}$. And

$$\frac{\partial \eta_B}{\partial X_B} - \frac{\partial \eta_{A'}}{\partial X_B} = \frac{(\eta_B - \eta_{A'})}{(1 + g_{A'} - g_B)}\left(\frac{\partial g_B}{\partial X_B} - \frac{\partial g_{A'}}{\partial X_B}\right)$$

5.7 Proof to Proposition 5

From (6) and (7),

$$\begin{aligned}
\frac{\partial^2\eta_{A'}}{\partial X_B^2} &= f_{A'}\frac{\partial^2\eta_{A'}}{\partial X_B^2}+g_{A'}\frac{\partial^2\eta_B}{\partial X_B^2} \\
&\quad +\left[\eta_{A'}\frac{\partial^2 f_{A'}}{\partial X_B^2}+\eta_B\frac{\partial^2 g_{A'}}{\partial X_B^2}+2\left(\frac{\partial f_{A'}}{\partial X_B}\frac{\partial\eta_{A'}}{\partial X_B}+\frac{\partial g_{A'}}{\partial X_B}\frac{\partial\eta_B}{\partial X_B}\right)+\frac{\partial^2 h_{A'}}{\partial X_B^2}\right] \\
&= f_{A'}\frac{\partial^2\eta_{A'}}{\partial X_B^2}+g_{A'}\frac{\partial^2\eta_B}{\partial X_B^2}+\left[(\eta_B-\eta_{A'})\frac{\partial^2 g_{A'}}{\partial X_B^2}+2\left(\frac{\partial\eta_B}{\partial X_B}-\frac{\partial\eta_{A'}}{\partial X_B}\right)\frac{\partial g_{A'}}{\partial X_B}\right] \\
\frac{\partial^2\eta_B}{\partial X_B^2} &= f_B\frac{\partial^2\eta_{A'}}{\partial X_B^2}+g_B\frac{\partial^2\eta_B}{\partial X_B^2} \\
&\quad +\left[\eta_{A'}\frac{\partial^2 f_B}{\partial X_B^2}+\eta_B\frac{\partial^2 g_B}{\partial X_B^2}+2\left(\frac{\partial f_B}{\partial X_B}\frac{\partial\eta_{A'}}{\partial X_B}+\frac{\partial g_B}{\partial X_B}\frac{\partial\eta_B}{\partial X_B}\right)+\frac{\partial^2 h_B}{\partial X_B^2}\right] \\
&= f_B\frac{\partial^2\eta_{A'}}{\partial X_B^2}+g_B\frac{\partial^2\eta_B}{\partial X_B^2}+\left[(\eta_B-\eta_{A'})\frac{\partial^2 g_B}{\partial X_B^2}+2\left(\frac{\partial\eta_B}{\partial X_B}-\frac{\partial\eta_{A'}}{\partial X_B}\right)\frac{\partial g_B}{\partial X_B}\right]
\end{aligned}$$

Substituting equation (**??**) into the above two equations and express them in the following way again

$$\frac{\partial^2\eta_{A'}}{\partial X_B^2} = f_{A'}\frac{\partial^2\eta_{A'}}{\partial X_B^2}+g_{A'}\frac{\partial^2\eta_B}{\partial X_B^2}+j_{A'} \tag{10}$$

$$\frac{\partial^2\eta_B}{\partial X_B^2} = f_B\frac{\partial^2\eta_{A'}}{\partial X_B^2}+g_B\frac{\partial^2\eta_B}{\partial X_B^2}+j_B \tag{11}$$

where

$$\begin{aligned}
j_{A'} &= (\eta_B-\eta_{A'})\left(\frac{\partial^2 g_{A'}}{\partial X_B^2}+\frac{2}{1+g_{A'}-g_B}\frac{\partial g_{A'}}{\partial X_B}\right) \\
j_B &= (\eta_B-\eta_{A'})\left(\frac{\partial^2 g_B}{\partial X_B^2}+\frac{2}{1+g_{A'}-g_B}\frac{\partial g_B}{\partial X_B}\right)
\end{aligned}$$

With an analysis analogous to Appendix 5.5 again, we have

$$\begin{aligned}
\frac{\partial^2\eta_{A'}}{\partial X_B^2}>0 &\iff (1-g_B)j_{A'}+g_{A'}\cdot j_B>0 \\
\frac{\partial^2\eta_B}{\partial X_B^2}>0 &\iff f_B\cdot j_{A'}+(1-f_{A'})j_B>0
\end{aligned}$$

The sign of $\frac{\partial^2\eta_{A'}}{\partial X_B^2}$ depends on the parameters $\left\{\tilde{T}_{A'},\tilde{T}_B,\Omega,X_B\right\}$ and the sign of $\frac{\partial^2\eta_B}{\partial X_B^2}$ depends on the parameters $\left\{\tilde{T}_{A'},\tilde{T}_B,\Omega,M^d,X_B\right\}$.

When $1<\tilde{T}_B<\tilde{T}_{A'}$ and $0<X_B<\frac{\sqrt{\tilde{T}_{A'}\tilde{T}_B}}{1+\sqrt{\tilde{T}_{A'}\tilde{T}_B}}$

$$\frac{\partial^2 g_{A'}}{\partial X_B^2} = \frac{2\tilde{T}_{A'}(\tilde{T}_{A'}-1)}{[\tilde{T}_{A'}(1-X_B)+X_B]^3} > 0$$
$$\frac{\partial^2 g_B}{\partial X_B^2} = \frac{2\tilde{T}_B(\tilde{T}_B-1)}{[\tilde{T}_B(1-X_B)+X_B]^3} > 0$$

According to Proposition 4, we also have

$$\left(\frac{\partial g_B}{\partial X_B} - \frac{\partial g_{A'}}{\partial X_B}\right)\frac{\partial g_{A'}}{\partial X_B} > 0$$
$$\left(\frac{\partial g_B}{\partial X_B} - \frac{\partial g_{A'}}{\partial X_B}\right)\frac{\partial g_B}{\partial X_B} > 0$$

Then $j_{A'} > 0$ and $j_B > 0$. Considering that $0 < g_{A'} < 1$, $0 < g_B < 1$, $0 < f_{A'} < 1$ and $0 < f_B < 1$, $\frac{\partial^2 \eta_{A'}}{\partial X_B^2} > 0$ and $\frac{\partial^2 \eta_B}{\partial X_B^2} > 0$.

5.8 Proof to Corollary 1

Just apply the proof to Proposition 5 to the symmetric case.

5.9 Scenario of endogenous $\widetilde{T}_{A'}$ and $\widetilde{T}_B$

In this subsection we assume $\widetilde{T}_{A'}$ and $\widetilde{T}_B$ are decreasing functions of X_B as larger countrie have relatively lower trade costs and larger benefit of established network externality. Besides, we make another two assumptions: (1) we assume that $\frac{\partial^2 \widetilde{T}_{A'}}{\partial X_B^2} = 0$, and $\frac{\partial^2 \widetilde{T}_B}{\partial X_B^2} = 0$ as a benchmark case, as there has been no empirical or theoretical evidence on the second order effect. (2) the symmetry between country A' and B still applies. That is, when $X_B = \frac{1}{2}$, $\widetilde{T}_{A'} = \widetilde{T}_B = 1$. Thus the functional form of $\widetilde{T}_{A'}$ and $\widetilde{T}_B$ are simply $\widetilde{T}_{A'} = 1 + \frac{t_{A'}}{2} - t_{A'}X_B$ and $\widetilde{T}_B = 1 + \frac{t_B}{2} - t_B X_B$ where $t_{A'} > 0$ and $t_B > 0$.

The proof to Proposition 5 still applies, except for new expressions of $\frac{\partial g_{A'}}{\partial X_B}, \frac{\partial g_B}{\partial X_B}, \frac{\partial^2 g_{A'}}{\partial X_B^2}$ and $\frac{\partial^2 g_B}{\partial X_B^2}$. But one can easily verify that when $0 < X_B < \frac{1}{2}, \frac{\partial g_{A'}}{\partial X_B} > 0, \frac{\partial g_B}{\partial X_B} > 0$, $\frac{\partial^2 g_{A'}}{\partial X_B^2} > 0$ and $\frac{\partial^2 g_B}{\partial X_B^2} > 0$. Therefore a conclusion similar to Proposition 5 still holds.

5.10 Tables

Table 1 Euro Share in Export Invoicing

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\gamma^t_{ea,i}$	1.172***	1.228***	1.063***	1.063***	1.152***	1.115***	1.115***
	(0.064)	(0.061)	(0.066)	(0.066)	(0.060)	(0.061)	(0.067)
$\gamma^t_{ea,i} \times \frac{Y^t_i}{Y^t_{ea}}$		-3.649***	-3.868***	-3.881***	-3.186***	-1.852**	-2.214***
		(0.691)	(0.653)	(0.724)	(0.643)	(0.831)	(0.697)
EU			0.133***	0.133***	0.0960***	0.0793***	0.101***
			(0.026)	(0.026)	(0.023)	(0.024)	(0.028)
$Heuro$				0.00136	-0.0197	-0.0187	0.0249
				(0.034)	(0.030)	(0.029)	(0.032)
$year_{euro}$					0.0230***	0.0269***	0.0308***
					(0.003)	(0.003)	(0.003)
$year_{euro} \times \frac{Y^t_i}{Y^t_{ea}}$						-0.225**	-0.277***
						(0.090)	(0.077)
$pip(euro - dollar)^t_i$							-0.694
							(1.764)
N	210	210	210	210	210	210	144
$adj.R^2$	0.616	0.660	0.697	0.696	0.765	0.771	0.872

Standard errors in parentheses

* $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table 2 Dollar Share in Export Invoicing

	(1)	(2)	(3)	(4)	(5)	(6)
$\gamma_{us,i}^{t}$	2.954***	3.361***	2.193***	2.334***	2.334***	2.666***
	(0.258)	(0.256)	(0.294)	(0.300)	(0.306)	(0.313)
$\gamma_{us,i}^{t} \times \frac{Y_i^t}{Y_{us}^t}$		-14.43***	-7.988***	-10.79***	-10.79***	-24.40***
		(3.039)	(2.899)	(3.221)	(3.238)	(5.133)
EU			-0.209***	-0.202***	-0.202***	-0.181***
			(0.033)	(0.033)	(0.033)	(0.033)
$Heuro$				0.0774*	0.0774*	0.111***
				(0.040)	(0.040)	(0.040)
$year_{euro}$					-0.0000119	-0.0133*
					(0.006)	(0.007)
$year_{euro} \times \frac{Y_i^t}{Y_{us}^t}$						0.360***
						(0.108)
N	159	159	159	159	159	159
$adj.R^2$	.452	.518	.615	.622	.619	.643

Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

Table 3 Euro Share in Import Invoicing

	(1)	(2)	(3)	(4)	(5)	(6)
$\gamma^t_{ea,i}$	1.264***	1.317***	1.222***	1.234***	1.277***	1.226***
	(0.069)	(0.063)	(0.079)	(0.082)	(0.080)	(0.081)
$\gamma^t_{ea,i} \times \frac{Y^t_i}{Y^t_{ea}}$		-4.278***	-4.351***	-4.166***	-3.633***	-1.810*
		(0.678)	(0.674)	(0.758)	(0.745)	(0.990)
EU			0.0526**	0.0502*	0.0326	0.0147
			(0.026)	(0.027)	(0.026)	(0.027)
$Heuro$				-0.0173	-0.0334	-0.0326
				(0.032)	(0.031)	(0.031)
$year_{euro}$					0.0117***	0.0164***
					(0.003)	(0.003)
$year_{euro} \times \frac{Y^t_i}{Y^t_{ea}}$						-0.260***
						(0.095)
N	199	199	199	199	199	199
$adj.R^2$	.63	.691	.695	.694	.715	.724

Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

Table 4 Dollar Share in Import Invoicing

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\gamma^t_{us,i}$	2.239***	3.061***	2.034***	2.062***	1.991***	2.213***	2.721***
	(0.293)	(0.344)	(0.408)	(0.419)	(0.433)	(0.446)	(0.473)
$\gamma^t_{us,i} \times \frac{Y^t_i}{Y^t_{us}}$		-15.04***	-8.498**	-9.081**	-8.589**	-17.69***	-25.58***
		(3.679)	(3.822)	(4.255)	(4.326)	(6.580)	(6.627)
EU			-0.135***	-0.135***	-0.139***	-0.135***	-0.0265
			(0.032)	(0.032)	(0.033)	(0.033)	(0.058)
$Heuro$				0.0125	0.0120	0.0299	0.0385
				(0.040)	(0.040)	(0.041)	(0.040)
$year_{euro}$					-0.00407	-0.0111	0.0106
					(0.006)	(0.007)	(0.009)
$year_{euro} \times \frac{Y^t_i}{Y^t_{us}}$						0.202*	
						(0.111)	
$year_{euro} \times EU$							-0.0325**
							(0.013)
$year_{euro} \times EU \times \frac{Y^t_i}{Y^t_{us}}$							0.353***
							(0.111)
N	150	150	150	150	150	150	150
$adj.R^2$	.278	.348	.413	.41	.408	.417	.445

Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

Table 5 Tipping Phenomenon in Export Invoicing of Thailand

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	share	share (logit)	share (logit)	share (logit)	share	share (logit)	share	share (logit)
x^t_{issuer}	2.94***	26.1***	2.17***	24.1***	1.89***	22.0***	2.72***	25.7***
	(0.152)	(1.08)	(0.269)	(1.72)	(0.265)	(1.67)	(0.248)	(1.68)
$\gamma^t_{partner,issuer}$			0.134***	2.65***	0.163***	2.87**	0.564***	4.66***
			(0.0407)	(0.261)	(0.0397)	(0.251)	(0.0538)	(0.364)
$\gamma^t_{Thailand,issuer}$			1.76***	6.62**	2.63**	13.2***	0.824*	5.19
			(0.479)	(3.07)	(0.493)	(3.12)	(0.472)	(3.20)
$pip(cur - baht)^t$					-0.370**	-2.79***	-0.270***	-2.35***
					(0.0734)	(0.464)	(0.0655)	(0.443)
$regime_{partner,issuer}$							-0.352***	-1.56***
							(0.0358)	(0.242)
N	338	338	338	338	338	338	338	338
$adj.R^2$	.527	.633	.552	.718	.583	.745	.676	.773

Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

Table 6 Tipping Phenomenon in Import Invoicing of Thailand

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	share	share (logit)	share	share (logit)	share	share (logit)	share	share (logit)
x^t_{issuer}	2.40***	25.9***	2.44***	25.6***	2.44***	25.7***	2.53***	25.9***
	(0.0997)	(0.768)	(0.101)	(1.07)	(0.100)	(0.622)	(0.0980)	(0.628)
$\gamma^t_{partner,issuer}$			0.191***	3.20***	0.193***	3.22***	0.449***	3.79***
			(0.0322)	(0.204)	(0.0320)	(0.199)	(0.0557)	(0.357)
$\gamma^t_{Thailand,issuer}$			-0.463**	-2.45**	-0.413**	-1.75	-0.462**	-1.86
			(0.187)	(1.18)	(0.188)	(1.16)	(0.181)	(1.16)
$pip(cur - baht)^t$					-0.119**	-1.65***	-0.125**	-1.66**
					(0.0556)	(0.345)	(0.0537)	(0.344)
$regime_{partner,issuer}$							-0.188***	-0.417*
							(0.0341)	(0.218)
N	400	400	400	400	400	400	400	400
$adj.R^2$	.591	.740	.625	.839	.629	.847	.655	.848

Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

Table 7 “Tipping phenomenon” with quadratic and cubic forms

	(1)	(2)	(3)	(4)
	export invoicing	export invoicing	import invoicing	import invoicing
$x^t_{issuer,i}$	-1.31**	-31.6***	-1.62**	-32.5***
	(0.728)	(4.74)	(0.652)	(4.94)
$(x^t_{issuer,i})^2$	11.6***	184***	11.7***	191***
	(1.98)	(26.7)	(1.82)	(28.5)
$(x^t_{issuer,i})^3$		-300***		-300***
		(46.4)		(47.7)
$\gamma^t_{partner,issuer}$	0.475	0.397***	0.356***	0.305***
	(0.054)	(0.052)	(0.055)	(0.053)
$\gamma^t_{Thailand,issuer}$	0.640	5.12***	-0.00441	2.27***
	(0.451)	(0.814)	(0.187)	(0.403)
$pip(cur - baht)^t$	-0.410***	-0.442***	-0.287***	-0.270***
	(0.067)	(0.063)	(0.057)	(0.055)
$regime^t_{partner,issuer}$	-0.219***	-0.0913**	-0.042	0.0402
	(0.041)	(0.0434)	(0.040)	(0.040)
N	338	338	400	400
$adj.R^2$	0.705	0.737	0.686	0.715

Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

5.11 Figures

The following graph verifies Proposition 3 from the import invoicing currency side.

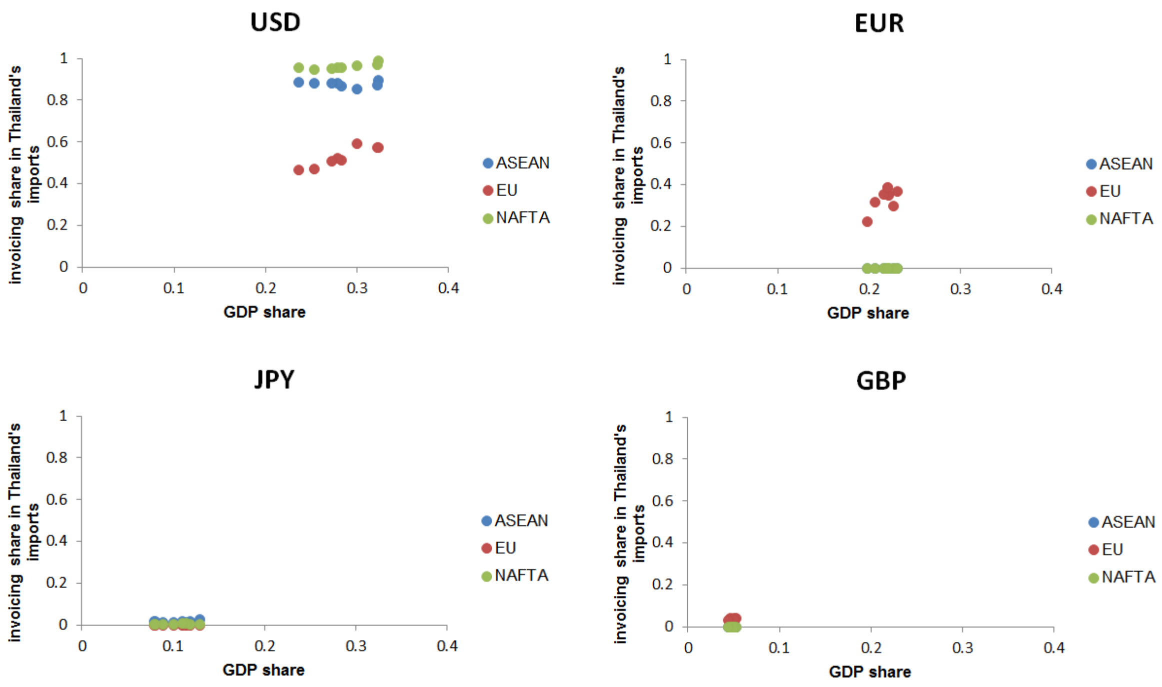

Figure 7 Four international currencies: import invoicing share to ASEAN/EU/NAFTA versus its home country's GDP share